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What Does It Mean To Preservice Mathematics Teachers To Anticipate Student Responses?

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WHAT DOES IT MEAN TO PRESERVICE MATHEMATICS TEACHERS

TO ANTICIPATE STUDENT RESPONSES?

by

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ABSTRACT

WHAT DOES IT MEAN TO PRESERVICE MATHEMATICS TEACHERS

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Lesson study is a form of professional development for teachers adopted in recent years from Japan. Introducing lesson study to U.S. teachers and researchers has been the focus of most of the literature on this subject. Much of the literature outlines how lesson study works and describes its essential features. One of the features of lesson study is anticipating student responses, also known as anticipating student thinking. Anticipating student responses is passingly described in lesson study literature. This research was conducted to understand what it means to anticipate student responses for preservice mathematics teachers in a lesson study group.

Lesson study literature indicates that anticipating student responses is to anticipate conceptual development from the students' perspective, and the purpose is to be prepared to have meaningful discussions and questions to enable students to develop the

understanding. Anticipating student responses is highly related to the hypothetical learning trajectory described by Simon (1995), the self directed anticipative learning model described by Christensen and Hooker (2000) and the expert blind spot discussed by Nathan and Petrosino (2003). While their work does not stem from lesson study, they add theoretical perspective to the idea of anticipating student responses. Their work indicates that anticipating student responses is difficult, valuable, that one gets better at it through experience, and that it is very useful in refining lessons.

Participants were enrolled in the mathematics education methods class of a large private university in the U.S. A characterization of anticipating student responses was developed as the participants met in group meetings to create a lesson. They anticipated student responses in ways that facilitated lesson planning and task design. Participants did not anticipate student responses toward students' conceptual development. This research reports five particular ways that anticipating student responses was used as a tool to define and refine the lesson so that it ran smoothly toward lesson goals. These ways are related to: goals, tasks and materials, procedural mathematical reasoning, successful student efforts, and emotional responses. It is believed that anticipating student responses towards task design is a necessary precursor to anticipating student responses toward students' conceptual development.

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Introduction

A teacher's obligation is to teach, which Robert Leamnson (1999) defines as "any activity that has the conscious intention of, and potential for, facilitating learning in another" (p. 3). The responsibility of learning rests squarely on the shoulders or the heads of the students. Though a teacher does all within his or her power to promote learning, since that is all they can do, that learning can only occur in the student's brain. Leamnson argues that "the thoughts that are important are those that go on in the heads of the students" (p. 99). By this, he means that no matter how well organized or clear a subject is in the teacher's mind, the point of education is to help students come to see the subject just as clearly. The usefulness of separating the teacher's duty to teach from the student's duty to learn is that one can then focus more specifically on the function and obligations of each party. Therefore, we are able to focus specifically on what the teacher can do to better promote learning among the students. Leamnson makes it clear that if one's intention is to teach, then one must be concerned with how the students will learn, or what will prompt them? What questions will promote thought and yield a desired outcome?

"Little (1999) noted that the 'systematic, sustained study of student work, coupled with individual and collective efforts to figure out how that work results from the practices and choices of teaching' may be one of the most powerful sites for teacher inquiry" (in Steinburg, Empson, & Carpenter, 2004, p. 239). Several other studies, (Ambrose, 2004; Empson & Junk, 2004; Franke, Carpenter, Levi, & Fennema, 2001; Steinburg et al., 2004) substantiate that when teachers begin to seriously examine student thinking as a part of their practice their teaching practice changes dramatically. Frank et al. (2001) found that teachers who engaged in practical inquiry into children's thinking were likely to generate their own change toward teaching that supported and enhanced students' understanding.

Participation in a form of Japanese professional development known as l*esson study* presents opportunities for teachers to think carefully and critically about student thinking. Lesson study has increasingly received attention in the last decade by U. S. researchers. It has particularly drawn attention since Stigler and Hiebert (1999) described it in their book *The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom.* Lesson Study gives teachers an opportunity to collaborate and provides structure and purpose to their meetings. In short, lesson study is made up of the cycle of collaboratively preparing a lesson, teaching the lesson and observing the student responses during the lesson, and finally, reviewing the student responses due to the lesson and revising the lesson to be taught again. This cycle allows teachers the opportunity to carefully investigate student responses and the effectiveness of lessons that promote student thinking.

One of the components of lesson study is *anticipating student responses* (Lewis, 2002a; Ready, 2002); it is also referred to in the literature as *expected student reactions* (Fernandez & Yoshida, 2004), or *anticipated student thinking* (Fernandez & Yoshida, 2004; Lewis, Perry, & Hurd, 2004) and will hereafter be referred to as anticipated student responses or (ASR). ASR is an inclusive phrase that indicates student's verbal, emotional, and cognitive responses to a lesson. For this reason anticipated student thinking can be thought of as a subset of ASR. The literature contains numerous references to ASR, but very little explicit information or description is given. It is thought that ASR, in the context of lesson study, provides lesson researchers opportunities to develop their own mathematical understanding, and is the means to begin to understand student thinking (Chokshi & Fernandez, 2004; Lewis, 2000; Lewis, 2002b; Lewis, Perry, & Hurd, 2004; Ready, 2002). It is also hypothesized that a careful examination of student thinking may change teachers' beliefs about content, teaching, and students abilities and understanding (Ambrose, 2004; Empson & Junk, 2004; Steinburg et al., 2004).

The lesson study model has been used since 2000 in the secondary mathematics education methods class of a large private university. A pilot study was conducted in the spring of 2004 in this methods course to examine the influence of discussing ASR on group discussions and lesson plans. A barrier that revealed itself in examining the affect of ASR was that it was not clear what ASR meant to the participants or how they were using it. It was determined that the affect of ASR could not be studied until it was understood what ASR meant to the participants.

Describing how pre-service mathematics teachers anticipate student responses in a lesson study group is the goal of this research. Terry Wood in an editorial of the Journal of Mathematics Teacher Education (2004) said, "these ideas [about student thinking] are still problematic… because the meaning of, for example 'understanding students' thinking', has not been discussed and common meaning fleshed out within the mathematics education community" (p. 173). The purpose of this research is to answer a part of the problem by characterizing ASR in the context of preservice mathematics teachers operating in a lesson study model. In other words, what things do they anticipate, and for what purpose do they anticipate student responses? By doing this, some common meaning will be given for how teachers anticipate and therefore understand student responses and thinking.

Conceptual Framework and Literature Review

This chapter will comprise three sections. First, studies of teacher's knowledge of student thinking will be discussed. Second, major components of lesson study, particularly those related to ASR will be described. Third, literature from non-lesson study perspectives will be reviewed and theories that closely relate to ASR will be discussed to provide several different perspectives.

Student Thinking in General

Much research has been conducted in an effort to increase teachers' awareness, value, and use of student thinking. A common method to achieve this goal is to engage teachers in practical or school based inquiry into children's thinking where teachers reflect on their own experience enabled through professional development courses and follow-up trainer visits (Empson & Junk, 2004; Franke, Carpenter, Levi & Fennema, 2001; Steinburg, Empson & Carpenter, 2004). One other program to be mentioned is that of a preservice mathematics teacher program where preservice teachers were given experiences teaching 10 year olds about fractions (Ambrose, 2004). These research programs attempt to improve teachers' abilities to understand and use student thinking in ways similar to lesson study.

Common among all of these studies is the general underlying effort to train teachers to teach math with a conceptual as opposed to a procedural viewpoint. A procedural understanding of mathematics may be described as superficial in that what is learned is not how or why things work, but what steps to follow when doing mathematics. Teaching mathematics procedurally is an effort to train students to follow procedures that are deemed necessary to solve problems. A student exhibits procedural understanding when they can follow a procedure correctly and produce right answers. Conceptual teaching is focused on helping students learn why and how mathematical concepts work and letting correct procedures emerge naturally as students make sense of the problem situation. While neither procedural nor conceptual philosophies necessarily dictate the method of teaching, it is commonly viewed that procedural teaching is a top-down, teacher-centered process where understanding is transferred to the students through the teacher's lectures and procedurally oriented assignments and activities. Conceptual teaching is commonly thought of as a bottom-up, student-centered process where ideas are produced by the students and the teacher works to create the environment and scaffolding that enables students to learn.

Empson and Junk (2004) conducted research in a district that was implementing a student-centered investigative curriculum, *Investigations in Number, Data, and Space*, (TERC, 1995-1998). They interviewed elementary teachers with a focus on "how teachers made sense of students' nonstandard strategies for multidigit operations" (p. 125). Their goal was to understand the depth and breadth of teachers understanding of students' understanding. They found that although the new curriculum did not move all teachers to value, examine, or learn from their students' thinking, some did come to this perception. Empson and Junk hypothesized that, "had the teachers had opportunities to listen to and discuss each other's responses in a formally organized school-based forum, teachers' understanding could have been measurably enhanced" (p. 139). They asserted that a curriculum alone is not enough to heighten teachers knowledge of student thinking and that "formal school-based structures [need to be] put in place" (p. 140). Lesson study is one of the school-based structures they suggest may be helpful to provide a formal structured environment where student thinking can be researched.

Franke et al. (2001) conducted a three-year professional development program designed to train teachers to use *cognitively guided instruction* (CGI). Their research was more aggressive than Empson and Junk's (2004), in that they wanted to do more than measure the depth and breadth of teacher's understanding of student thinking, they wanted to promote it. CGI provides a framework for teachers to describe, analyze, and discuss student thinking. During the three-year professional development phase of their research they determined different levels of engaging student thinking exhibited by the teachers. They followed up three years later in an effort to find how or if teachers were continuing to examine student thinking, what level they were then at, and what factors lead to growth in teachers' understanding of students' conceptual development. They developed a table that integrated beliefs and practices of teachers relative to their engagement with student thinking (see Table 1).

Table 1

Levels of Engagement with Children's Mathematical Thinking

Level 1: The teacher does not believe that the students in his or her classroom can solve problems unless they have been taught how.

- Does not provide opportunities for solving problems.
- Does not ask the children how they solved problems.
- Does not use children's mathematical thinking in making instructional decisions.

Level 2: A shift occurs as the teachers begin to view children as bringing mathematical knowledge to learning situations.

- Believes that children can solve problems without being explicitly taught a strategy.
- Talks about the value of a variety of solutions and expands the types of problems they use.
- Is inconsistent in beliefs and practices related to showing children how to solve problems.
- Issues other than children's thinking drive the selection of problems and activities.

Level 3: The teacher believes it is beneficial for children to solve problems in their own ways because their own ways make more sense to them and the teachers want the children to understand what they are doing.

- Provides a variety of different problems for children to solve.
- Provides an opportunity for the children to discuss their solutions.
- Listens to the children talk about their thinking.

Level 4A: The teacher believes that children's mathematical thinking should determine the evolution of the curriculum and the ways in which the teachers individually interact with the students.

- Provides opportunities for children to solve problems and elicits their thinking.
- Describes in detail individual children's mathematical thinking.
- Uses knowledge of thinking of children as a group to make instructional decisions.

Level 4B: The teacher knows how what an individual child knows fits in with how children's mathematical understanding develops.

- Creates opportunities to build on children's mathematical thinking.

- Describes in detail individual children's mathematical thinking.
- Uses what he or she learns about individual students' mathematical thinking to drive instruction.

Note. From "Capturing Teachers' Generative Change: A Follow-Up Study of Professional Development in Mathematics," by M. L. Franke, T. P. Carpenter, L. Levi, and E. Fennema, 2001, *American Educational Research Journal*, *38(3),* 653-689. Copyright 2001 by the American Educational Research Association; reproduced with permission of the publisher.

A curious finding was that teachers at level 4b were not constrained in their detail of students' thinking by the cognitively guided framework but elaborated on it. They created this higher level of detail by engaging in practical inquiry and continued to test and revise their knowledge about student thinking which resonates with the framework of lesson study. This engagement with students' thinking was described as generative because teachers took it upon themselves to understand more deeply students' conceptual development. They were able to learn from their students, unlike teachers at level 3 who listened to their students but did not use what they heard to come to greater understanding of children's mathematical development. Franke et al. (2001) concluded that "focusing on student thinking proved to be a valuable mechanism for engaging teachers in generative change" (p. 685). The engagement with children's mathematical thinking that is described in the lesson study literature is level 4b.

Steinburg et al. (2004) took a different approach from Franke et al. (2001) to encouraging teachers to engage student thinking. They observed and interviewed one teacher over a period of five months with a focus on the teacher's knowledge of and efforts to build on, children's thinking and her decision making process relative to the same. During the five month observations and interview segment the teacher's teaching methods changed to be more student-centered. The following year the teacher was observed in order to assess whether she maintained the changes in her practice, and was finally interviewed several years later to determine what events spurred her growth as a teacher. They also used Table 1 as developed by Franke et al. (2001), to identify the level of engagement with children's mathematical thinking of this teacher.

Steinburg et al. (2004) found that the teacher moved from a level 3 to level 4a which are different in that level 3 teachers provide students opportunities to listen to one another and they provide different types of problems to the students, level 4 teachers know greater detail about student's mathematical thinking and use that knowledge to make instructional decisions. They felt that the interviews stimulated her change. The interviewer did not attempt to coach the teacher to operate at level 4a, but by asking specific questions about student's thinking that the teacher could not answer well, she was in some way directed toward the children's thinking that she did not fully understand and chose to investigate it on her own. This led the teacher to experiment in her classroom not only to understand in greater detail individual children's thinking, but to orchestrate class discussions to enhance the learning of all. "She attributed her growth in teaching to two main factors: a second pair of eyes in the classroom focused on children's thinking, and the freedom to experiment with instruction based on children's thinking" (p. 257).

Ambrose's (2004) study of preservice teachers' beliefs and her efforts to change such also provide background to this study. Research on beliefs and accompanying change in beliefs provides valuable perspectives in much of the research in mathematics teacher education performed today, especially in light of the efforts of reformers attempting to bring more conceptual type approaches to teaching mathematics. Although beliefs and change in beliefs is not the focus of this research, knowing typical beliefs of preservice mathematics teachers gives some indications as to the mindset of the participants in this research. Ambrose cited several sources in establishing that typical preservice mathematics teachers view teaching as telling. They hold caring attitudes toward students and hope to promote positive self-concepts among them. Ambrose claimed that as a result of her research, wherein preservice teachers had experiences attempting to teach fraction understanding to 10 year olds, the preservice teachers began to view teaching as a more difficult endeavor than they had previously believed and that teaching as telling was not as effective as they had expected. The preservice teachers had come to value giving students opportunities to think carefully and noticed that students understood concepts in ways different from their own and from each other. They had begun to notice student thinking in ways that changed their outlook on what it means to teach in ways that enhance student learning.

These studies examined teacher's knowledge of student thinking and found that when teachers were directed to study student thinking, their knowledge and beliefs about teaching and student learning changed. Critical examination of student thinking that was prompted by design from these studies is inherent in lesson study. By anticipating student responses and learning from student responses during lessons as components of lesson study, teachers learn how to design better lessons with how students learn in mind.

ASR and Lesson Study Components That Enable and Enhance ASR

Major components of lesson study to be examined are lesson goals, tasks, anticipated student responses, and observations. Lesson goals lead to choosing appropriate tasks, once tasks are chosen researchers anticipate student responses. Finally the lesson is taught and carefully observed to see if it meets the lesson goal, if the tasks were appropriate, and how students came to understand the concepts. Particular emphasis is given to the concept of anticipating student responses.

Implementing lesson study in the U. S. has been the focus of many papers and conferences (Chokshi & Fernandez 2004; Fernandez, Cannon, & Chokshi, 2003; Fernandez, Chokshi, Cannon, & Yoshida, 2001; Lewis, 2000; Lewis, Perry, & Hurd, 2004; Lewis, 2002a; Lewis, 2002b; Stigler & Hiebert,1998; Stigler & Hiebert, 1999). Yet there are many questions that require further investigation. What is it in the modus operandi of lesson study that heightens teachers' sensitivity to student thinking and learning? To answer this question it is necessary to elaborate on the cycle of preparing, teaching, and revising. It is also necessary to examine the critical components to which lesson study participants attend.

To begin with, lesson study is an iterative process of lesson design and pupil learning research. The lesson researchers, as those engaged in lesson study are often referred to, hypothesize that the lesson they develop, (the research lesson), will meet a set of well-defined goals. This hypothesis is refined as the lesson is taught, revised and taught again. In this way, lesson study is occasionally referred to in the literature, (e.g. Lewis, 2002b; National Research Council, 2002; Fernandez, et al. 2003) as loosely following the scientific method.

Lesson Goals

Research lessons are carefully designed to meet a set of very specific goals that educators have generated for the students. Having a clear set of goals enables the lesson researchers to anticipate student responses to better prepare the lesson. The goals are both general and specific, but as the lesson is created it is first designed to meet general goals of the school. They include overarching social and intellectual goals that educators have for their students. These types of goals are often created by the faculty of a school and are not unlike mission statements that many schools in the U. S. develop. The goals are very important to the lesson researchers and are generally brought about by considering the characteristics of an idealized student and contrasting that image with actual students. An example of an overarching goal might be *to encourage student's self-directed learning*, or *to teach students to listen carefully to one another's ideas*. Inside of these broad goals are specific content goals. Specific goals tend to focus on content that is known to be difficult to students or are central to a topic. Content goals are both general and specific.

Wiggins and McTighe (1998) suggested a *backwards* approach to lesson design that harmonizes with the structure of lesson study and has components similar to ASR. They suggested that goals are developed first. Goals are developed by identifying desired results and passing them through four filters. The four filters are:

> Filter 1. To what extent does the idea, topic or process represent a "big idea" having enduring value beyond the classroom?
>
> Filter 2. To what extent does the idea, topic or process reside at the heart of the discipline?
>
> Filter 3. To what extent does the idea, topic or process require uncoverage [or scaffolding]?
>
> Filter 4. To what extent does the idea, topic or process offer potential for engaging students? (p. 10, 11)

These desired results are somewhat like anticipated student responses in that they are outcomes that should be seen through the lesson; however, they are quite general.

After identifying desired results Wiggins and McTighe (1998) suggested that teachers define what will be accepted as evidence of student understanding and learning. This is much more like anticipated student responses from lesson study. Having identified desired results and what will be accepted as evidence of understanding or learning one can finally plan the lesson. This is in some ways backwards from what they might consider typical lesson planning where the textbook gives the lesson to be taught, then the teacher plans the goals to be achieved by it.

Defining acceptable evidence of student learning is much like ASR from the perspective of lesson study and like ASR does for lesson study so also does defining acceptable evidence provide a metric by which the lesson is measured. The difference is that ASR may be described as more inclusive in that lesson researchers are not only looking at evidence of learning, but also consider thought processes that lead to learning. Designing lessons with pre-conceived notions of evidence of students' understanding aids both the lesson design process, and the teaching and reflection on teaching that follows because teachers have ways of measuring whether they are meeting their goals.

Detailed Lesson Plan

In lesson study, once the goals are established and understood, a detailed lesson plan is generated. A detailed lesson plan serves several purposes: it communicates to other researchers the rationale for the lesson and apprises observers of the lesson particulars; it serves as a basis for discussion among lesson researchers involved in the lesson; it provides direction for the teacher in responding to student thinking and other logistical pieces. The full lesson plan document begins by discussing the goals, the necessary prerequisite knowledge, the content to be taught before and after the lesson, the present state of the student understanding in which the lesson will be taught, and the motivation for the particular lesson. The level of detail described here is extraordinary and provides the lesson researchers with “a good opportunity to think deeply about how students learn” (Fernandez & Yoshida, 2004, p. 48).

Included in this initial write-up is a description of the particular content to be learned. It is written so that all involved know what the focus is, and so that observers of the lesson and future researchers can know what the motivation was for the lesson. An example from Fernandez and Yoshida (2004) follows:

> In this lesson, the students will encounter subtraction problems (such as 10 to 19 minus 1 to 9) that cannot be solved without regrouping (i.e., by subtracting the number from the number in the ones position). Students will see that by using concepts learned in previous lessons, it is possible to solve these problems by taking the one from the ten’s position to make ten (i.e., regrouping). The students will realize that once this step is taken, they can proceed to solve the problem by using strategies they have learned in [the] past. In addition, this lesson hopes to deepen the student’s understanding of the 10 decimal system (place value). Furthermore, through this lesson, the students should be able to perform subtraction with regrouping by choosing the most efficient method given the numbers involved. (p. 36)

The level of detail in this paragraph is striking and even more impressive is that it is just one of seven paragraphs in a thorough description of the content to be learned, what lessons and concepts preceded, and what concepts will build upon this lesson. Having such depth of thought before writing the lesson in turn allows the actual lesson to be very precisely written. The detail indicated here enables researchers, both those within and without of the development of a particular lesson, to have a clear understanding of the purpose.

The lesson plan that one of the lesson researchers actually teaches from typically includes columns for the teacher's activities and questions, the anticipated student responses, points to notice and evaluate, and materials and strategies to employ given student responses (See Appendix A). Lesson plans written this way not only provide initial direction for the teacher, but also inform the teacher of possible student responses, how to respond in the case of each, and additionally how to evaluate the learning and progress of the students at particular points in the lesson.

Student thinking is of utmost importance in writing and revising lesson plans, but ideas about student thinking and lesson ideas come through experience of planning, teaching, and revising lessons. Catherine Lewis (2002b) writes that in lesson study the participants use many resources to help them plan the lesson. She writes, "Lesson study is most productive when educators build on the best existing lessons or approaches, rather than reinventing the wheel" (p. 62). Lewis then quotes a Japanese educator who said, "If you shoot for originality too early in your development as a teacher, you're likely to fail. Initially, you must take a lot from others" (p. 63). Chokshi and Fernandez (2004) harmonize on this point saying that lesson researchers often use resources such as different text books, other lesson plans and previous research lessons. These resources aid the participants greatly by giving them ideas for tasks and questions, ways to motivate the lesson, what to watch for to determine success, and potential student responses.

Tasks

On what are anticipated student responses based? Anticipated student responses to what? These two questions draw out that anticipated student responses must be responses to questions or tasks, or a lesson in general. Tasks and questions then become the domain in which anticipated student responses are developed. As just described, Japanese researchers use many resources to get ideas for tasks and questions. Since student thinking is so important to lesson researchers it is natural that the types of questions and tasks they use allow for different thinking. Implicit and explicit in the literature on lesson study are references to open-ended questions. Lesson researchers are careful to word their lesson questions so that the answers to them are not the only part that is valued in the classroom discussion. If an answer to a question or tasks is given without the thinking involved, the teacher is quick to follow-up with questions that elicit thinking (e.g. Fernandez & Yoshida, 2004, *hatsumon*).

In relation to ASR, *The Professional Standards for Teaching Mathematics* (National Council of Teachers of Mathematics [NCTM], 1991) emphasized the importance of "a*nalyzing* student learning, the mathematical tasks, and the environment in order to make ongoing instructional decisions" (p. 5). Hiebert et al. (1997) explained that "when selecting tasks or problems, we need to think ahead about the kinds of relationships that students might take with them from the experience" (p. 22). These references indicate that tasks are very important in learning environments and need to be carefully thought out in regard to their function, especially the thinking they promote from students.

Good tasks need to be implemented in such a manner that the cognitive demand required of the students is appropriate, not so hard that students quickly give up and not so easy that no learning is required (Arbaugh & Brown, 2004; Romberg & Kaput, 1999, Smith & Stein, 1998). Good tasks need to be aligned with our current understanding of how students learn (van Merriënboer & Paas, 2003; Simon, 1995). Tasks need to be structured in such a way that they build on student's thinking and make apparent students conceptual development so that teachers can build on their students understanding (National Research Council, 2000). Teachers need to be familiar with student responses and the natural progression of understanding so that they can be prepared to discuss or ask questions at the right moment giving students the ideas necessary to make sense of their struggle without taking the learning away from them (van Merriënboer & Paas, 2003; National Research Council, 2000; Stein & Smith, 1998). Tasks also need to be engaging and stimulate rich discussion (Lappan & Friel, 1993).

While the literature is rich with ideas about what ideal tasks should be and should do and how they ought to be launched and managed in the classroom, little is said of how to develop good tasks. The ideals of good tasks certainly serve as a guide in developing them (e.g. Smith & Stein, 1998), but how teachers work and struggle to create them is not described. As mentioned, lesson researchers do not necessarily design entirely new lessons, they gather resources that fit with what they need in their lesson and modify it to make it fit even better.

There are a lot of *shoulds* in the literature on tasks, but it is important to focus on the student responses that comes from them, and not on the tasks alone. Thinking about mathematical tasks and what students will do with them is a rich domain on which to reflect for math educators in learning about students' conceptual development (Stein & Smith, 1998). However, Ball (2000) points out in the foreword of *Implementing Standards-Based Mathematics Instruction: A Casebook for Professional Development*, that too often in professional development settings, discussion on tasks remains at the structural level, "asking whether learning is better in small groups or in whole class, debating the merits and perils of lectures, worrying about the quiet learner" (p. ix). A dichotomy surfaces in relation to tasks when preparing a lesson. One branch may deal with structural or organizational details of tasks and the lesson in general. The other branch may deal with learning about the conceptual development of students and using this knowledge to design better lessons.

The tie between tasks and ASR is very strong as described by Bromme and Juhl (1988). They write, "The teacher uses tasks to help pupils understand. In order to do this, information about the pupil and information about the task must be related to one another. Only this information enables him to plan and implement the next steps of teaching" (p. 275). In terms of lesson study, ASR is characterized as a means whereby teachers research student thinking and plan lessons that enhance students' understanding. ASR is an important part of the group discussion in the process of lesson study and much of the discussion about tasks is in terms of ASR. To emphasize the role of ASR in these lesson research discussions Shimizu (2002) said, "the anticipated students' responses make up a large part of the lesson plan" (p. 57).

Anticipated Student Responses

Lesson study focuses teachers on student responses which includes their thinking, which has been described as necessary for today's teachers. Two explicit ways that the structure of lesson study directs lesson researchers to student responses is, first in the working environment of planning the lesson and the nature of the participants discussions where anticipated student responses are generated, and secondly, in the lesson plan where the anticipated student responses and thinking are written. ASR is mentioned in much of the literature as something the lesson researchers do, but it is rarely elaborated and explained fully. Perhaps the lack of definition of ASR is due to the oversight of researchers. After all, it seems that the phrase is self-defining. Though ASR is not well defined in the literature, it is often discussed in the context of its purpose and function and here it is clear that ASR is a tool that lesson researchers use to think critically of the development of concepts. Like a screwdriver is difficult to describe apart from its function, so also is ASR.

*ASR as described by lesson study researchers. T*o define ASR we begin by asking, *what is it that teachers anticipate or what is the object of their anticipations*? Secondly, *for what purpose do researchers anticipate student responses?* Fernandez et al. (2003) provided meaning as they reported Japanese teachers supporting U. S. teachers in lesson study. The U. S. teachers had anticipated students' solutions to mathematical problems:

> However,… they did not discuss what these solutions conveyed about 1) how students understood the problems or 2) how to facilitate children's understanding. (p. 180)

> Clearly the Japanese teachers were trying to convey that to develop students' understanding, teachers need to consider not only what they want students to learn, but how they expect them to learn it and how they are going to help students reach that understanding. In particular, the Japanese teachers conveyed that the problems that teachers pose to students, as well as the precise way these problems are presented, affect how students will approach and think about the content of the lesson…. To have students produce anticipated solutions is not an end in and of itself… the Japanese teachers were more concerned with whether or not the students had actually understood the relationship between their solutions and the formula for area of a triangle presented at the end of the class. (p. 181)

This example demonstrates that to anticipate student responses is to anticipate the students' development of concepts. It then appears, that the object of ASR is the conceptual development from the students' perspective, and the purpose is to be prepared to have meaningful discussions and questions to enable students to develop the concepts.

One other lucid excerpt serves to elaborate the meaning and purpose of ASR. Fernandez and Yoshida (2004) near the end of their book *Lesson Study: A Japanese Approach to Improving Mathematics Teaching and Learning* write:

> It also makes sense that the Tsuta teachers spent a great deal of time putting themselves in the shoes of the students who would take part in their lesson. In thinking about the problem they would pose, they used what they knew about the children to design a problem that would

challenge them, attract their attention, and help them develop a better understanding of regrouping. In particular, they made every effort to pick a problem that students would solve in multiple ways. They made sure to anticipate all the possible student responses and solutions that might come up and to use these responses as a resource for their lesson planning. In fact, several of the teachers at Tsuta explained during their interviews that carefully anticipating student responses is the key to understanding students' thinking processes. They were also of the opinion that the study of students' anticipated answers and possible counteranswers prepares a teacher to lead a well-organized discussion that builds from student ideas. (p. 229)

These two examples together help clarify ASR. In addition to defining ASR as knowing how concepts develop in students' minds, it is implied that ASR takes into account students present knowledge, skills, and abilities and anticipating how those will be used in understanding new concepts. Also implied is what misconceptions students are likely to have, or what difficulties they might encounter given their current state of knowledge. Additionally, it is inferred that ASR prepares teachers to lead a class discussion. In reference to ASR, Diane Ready (2002) explained that "[by anticipating student responses beforehand], teachers do not need to be so relentlessly fast on their feet when a class takes a disorienting turn" (p. 4).

ASR is anticipating students' conceptual development, the likely paths and difficulties inherent in students' conceptual development, misconceptions, and emotional responses such as interest. The purpose of ASR is to know the experience from the students' perspective, to prepare an orchestration of ideas in the classroom, and to design a lesson that is meaningful and engaging to students. Some may claim that this definition is still inadequate, but these are the apparent facets as derived from the literature. If we liken learning to a journey, it may be said that to anticipate student responses is to prepare a map to guide all students to a destination. The map has multiple routes, each route having different advantages and disadvantages. To anticipate student responses is to take the journey from the students' perspective. In this analogy, the purpose of ASR is then to try to make the journey as meaningful as possible for the students in arriving at the destination. Perhaps this analogy provides a different reason for why ASR is not thoroughly described, because the telling of a journey cannot tell all. Usually only highlights are given, just as introductory literature to lesson study explains ASR as one component among many.

Other perspectives about anticipation. There are many passing references to the concept of ASR both in the literature on lesson study and in mathematics education research in general. While few explicit discussions on the concept occur, some provide additional perspective. It should be noted that it is commonly viewed that lesson study uses a constructivist epistemology and that the current reform movement is largely based on some form of constructivism. The important distinction here is that teaching as telling is not viewed as the way students learn, but that "understandings are constructed by learners as they attempt to make sense of their experiences, each learner bringing to bear a web of prior understandings, unique with respect to content and organization" (Simon & Schifter, 1993, p. 331). This implies that teachers cannot assume that their teaching is effective, but that teachers must be even more sensitive to students' conceptions as they develop. This dramatically complicates teachers' efforts. Perhaps this is one reason why lesson study and similar forms of professional development have received so much attention since they treat teaching as ongoing scientific research.

In this vein of redefining mathematics teaching practice, Simon (1995) suggested what he called the *hypothetical learning trajectory*, which he explained is made up of three components: "The learning goal that defines the direction, the learning activities, and the hypothetical learning process—a prediction of how the students' thinking and understanding will evolve in the context of the learning activities" (p. 136). These components correlate respectively to the goals, tasks, and anticipated student responses. He connects the hypothetical learning process or ASR and tasks by explaining that:

> The development of a hypothetical learning process and the development of the learning activities have a symbiotic relationship; the generation of ideas for learning activities is dependent on the teacher's hypotheses about the development of students' thinking and learning; further generation of hypotheses of student conceptual development depends on the nature of anticipated activities. (p. 136)

So he suggests developing tasks that are hypothesized to best enhance students' conceptual development. Then having such tasks, one can further hypothesize how students will learn from them. He goes on to state that the hypothetical learning trajectory aids the planning process of lessons and helps the teacher become sensitive to student difficulties and prepare to address them and furthermore prepares the teacher to be responsive and flexible in the classroom.

Simon's construct helps us address critical questions in mathematics teaching, namely, "How do our learners get from one level of understanding to a higher level?" In other words, how do we know that our student's understanding has increased? Students need to have experiences so that, "[their] mental comparison of the [experience(s)] allows for recognition of patterns, that is, abstraction of the relationship between activity and effect" (Simon et al, 2004, p. 319). Providing these experiences is a critical obligation for teachers and hypothesizing is a tool that is used to both produce lessons that enable students to gain understanding and modify unsuccessful lessons that did not help students understand (Simon & Tzur, 2004).

To hypothesize or anticipate effects of actions or constructs is natural to life. For example, if you know someone with a difficult personality you come to expect tension even though that person might change. Anticipating the tension with the person is useful in that it provides structure to your actions. In determining patterns and anticipating future events predictability is a comforting notion. Another example is when you flip a light switch, do you know that the light will come on? Or, do you anticipate that it will? And, if a light burns out do you stop flipping the switch because you know it is ineffective and may continue to ignore the switch after the light is replaced to your unawareness? The truth is that in any goal directed activity, hypotheses and anticipations naturally emerge. Why would we do research if the results were always bewilderingly random? In this light, the concept of ASR is a natural part of any lesson plan, but in lesson study, it is used aggressively for specific purposes.

The very first research lesson that a group gives will have all student responses based largely on hypotheses. Later teachings of the same lessons are not as unpredictable.

Many of the anticipated student responses from the first lesson are still anticipated for future lessons, and even more, they are anticipated because the researchers have actually seen them occur in response to their lesson. Through this iterative process, the anticipated student responses, while still hypothetical, are partially based on prior realities. In this way the lesson ensures to a great degree that students will work along paths that are largely predicted or planned. It is not to say that students will not forge a new path through the lesson, but that any new path is likely to be somewhat similar to previous solutions the teacher has seen. This gives the teacher the power to enhance and deepen the experiences for all of the students because he or she can provide experiences for the students to learn from each other.

Christensen and Hooker (2000) described anticipation as applied to learning in general and called it *self-directed anticipative learning*, which is described in the epistemology of adaptive interactionism. The essence of their work is that "self-directed systems anticipate and evaluate the interaction process and modulate system action accordingly, thus generating action contextsensitively in order to achieve specific interaction goals" (p. 14). They are saying that intelligent beings naturally anticipate outcomes and modify their efforts to achieve more successful outcomes. But, included in their work is further elaboration that indicates exponentially increasing ones' learning ability. In other words, when one has learned things through experience, one is enabled to learn more things because one has more ways of knowing, more experience to draw upon and hypothesize from. This implies that lesson researchers who have some experience teaching and researching lessons have greater capacity to anticipate students' cognitive development, but also greater capacity to see critical developments in the classroom. It is as the artist who has greater capacity to view great art over the abilities of the average passer-by because they have learned to see better. Christensen and Hooker say it this way:

> [Self-directed anticipative learning] processes gain increased power through the construction of anticipatory models of the interaction process. These anticipations generate new information by modifying interaction, and modification and enrichment of the anticipations results in improved ability to localize success and error, thereby improving learning capacity. (p. 20)

In viewing some of these more theoretical perspectives on the concept of anticipating or hypothesizing several points emerge. First, it seems that anticipating outcomes is fairly natural, even subconscious. We anticipate everyday actions and outcomes and regulate our plans or schedules around what we anticipate. Second, anticipating has been highlighted as a part of the learning process by both constructivists and adaptive interactionists. By anticipating outcomes we adapt or learn to do things differently to be more successful at tasks and problems presented by life. Third, anticipating is integral with the planning process of activities, especially lessons. We make specific lesson plans because we either consciously or subconsciously anticipate that they will enhance student learning. Fourth, the ability to anticipate well is increased through experience and in tandem with this, fifth, the ability to see experiences with enlightened eyes is increased because we know what to look for. Hence, where anticipation is an almost subconscious mechanism in people's everyday lives, lesson study makes it a conscious effort, which enhances lesson planning which in turn enhances student learning.

One final theoretical lens that harmonizes with anticipation but comes from a different direction is the phenomenon known as the *expert blind spot*. The essence of this idea is that people with great knowledge of a topic tend to overlook the difficulty it takes to gain such knowledge and they organize the information in ways different from the ways people learn. Nathan and Petrosino (2003) concluded that:

> Educators with advanced subject-matter knowledge of a scholarly discipline tend to use the powerful organizing principles, formalisms, and methods of analysis that serve as the foundation of that discipline as guiding principles for their students' conceptual development and instruction, rather than being guided by the knowledge of the learning needs and developmental profiles of novices. (p. 906)

They caution that this claim does not imply that subject-matter knowledge is destructive for teaching; after all, how would one teach without knowledge of the subject? They do suggest, however, that teachers prepare their lessons in view of how novices learn the subject. They call this type of knowledge pedagogical content knowledge. They define expert teachers as those that, "organize their knowledge of subject matter, students, and pedagogy more deeply, in ways that readily facilitate lesson planning and teaching" (p. 907). Thus, it is requisite that teachers have both deep subject matter knowledge and pedagogical content knowledge which they say, "appears to develop out of classroom teaching experiences that also draw on subject-matter knowledge" (p. 908).

Nathan and Petrosino (2003) explained that those with advanced subject-matter knowledge underestimate the difficulty that novice learners have in learning. Another phrase for this expert blind spot is the "curse of knowledge." Research in this area shows that, "adults and children who know the solution to a problem tend to overestimate how easy it is for someone else to solve that problem" (p. 919). In their research it was found that, "participants relied largely on impressions in making their difficulty ratings, rather than on the more deliberative, analytical method evident among participants who judged difficulty but did not solve the problems" (p. 919). Whether participants in this study are likely to rely on impressions or an analytical method, it is not known. Nevertheless, they may have an expert blind spot because of the number of math content courses they have taken.

Observation Component

When a member of the lesson study group (LSG) teaches the lesson, the other members of the LSG and other teachers carefully observe. The focus of their observations is directed by the hypothesis that the lesson will meet the goals outlined in the lesson plan. This indicates that great attention should be given to how students actually learn. Lesson study researchers observe the reaction and attitude of the students to the lesson, and the students learning and development. Sometimes, observers are given additional lenses through which to observe the lesson, for example to pay attention to the thinking and reaction to the lesson by particular students. Lewis et al. (2004) write:

> Team members collect specific data, which generally include detailed narrative records of the learning of several students—what the students said and wrote, how the students used the materials, what specific supports

encouraged understanding, and what obstacles to learning arose during the lesson. (p. 20)

The rigor and focus of this observational component is made clear by a response from Japanese peers to teachers in the U. S. who were learning about lesson study. The U. S. teachers prepared and taught a lesson and generally felt that they had met all of their goals, but they did not gather or provide specific evidence that their lesson had been successful. "Japanese teachers stressed [the need] to think carefully about what evidence was required to claim that a lesson had achieved its objectives" (Fernandez, et al. 2003, p. 175). Fernandez et al. (2003) further commented about the U. S. teachers' second lesson, "They did not conduct this extra lesson with any particular research question in mind, such as which factors determine why children solve problems in certain ways" (p. 175). In other words, the U. S. teachers taught a research lesson without scrutinizing the results or carefully examining how students learned.

Lesson study helps teachers to think through lessons from their students' perspectives. It is because of the careful attention to such details, from the lesson goals and content to be learned, to the planning process including ASR, to the lesson as conducted and the observations of students, that teachers come to know the affect of their lesson through the students' eyes. Observing the lesson from the student perspective is critical because it is the data collection necessary to evaluate the success of the lesson. Catherine Lewis (2002b) writes that:

> The gold standard for judging the research lesson is student learning and development…. Lesson study…provides a means for teachers to develop their evidence-gathering skills and ability to see a lesson from the

student's point of view. Developing 'the eyes to see children' is, in the view of many Japanese educators, the most important goal of lesson study. (p. 36)

The observational component of lesson study also provides some insight into ASR. Japanese lesson researchers describe that observers look not only for student thinking but engagement, excitement, and "'shining eyes'" (Lewis, 2002b, p. 66). If these are some of the things that are looked for in most research lessons, then it indicates that they are things lesson researchers anticipate. Descriptions of teachers planning a lesson in Japan in Fernandez and Yoshida (2004) tell how teachers anticipate the emotional interest of the students. In particular they modify the lesson design so that it is more likely to engage students.

Worth mentioning is a study reported by Laura van Zoest (1995), which was designed to determine preservice mathematics teachers' focus while observing mathematics instruction. Since the focus of observations may indicate what was anticipated, this study may accordingly indicate what general types of things that the participants in this study may anticipate. She listed the five most significant observations made by preservice teachers in terms of time and attention. They were: a) classroom management strategies, b) instructional strategies, c) teaching style, d) student behaviors, and e) mathematics content" (p. 4). This is in sharp contrast to the observational component of lesson study, where observing student thinking is paramount. Perhaps these specific observations and their order of importance are natural to preservice teachers because they are not trained in what to observe that is more important, and maybe it is because preservice teachers will soon be filling the shoes of those they observe and they can see themselves doing so.

Reflection Component

After the lesson is taught a reflection meeting is held to discuss what was observed, how and whether the lesson met the goals, and ways that the lesson could be improved. The lesson researchers will continue reflecting on the lesson as it was taught as they revise and improve it. The lesson will then be taught and observed again followed by another reflection meeting.

Research lessons are taught at least twice and occasionally are taught three or more times. A final write-up of the lesson research includes the evolution of the lesson. The process of lesson study research can be quite short, spanning as few as three weeks. Generally, the second teaching of a lesson closely follows the first teaching. The process of lesson study could go on indefinitely, but as the literature suggests, there are diminishing returns for teaching a lesson many times, and the purpose is not to create a perfect lesson but to learn from the process. Lesson study is valued by participants for several reasons including: the opportunity to work collaboratively with others, the opportunity to reflect on one's teaching, and the opportunity to think deeply about and observe carefully student learning among others. A Japanese lesson researcher reported to Catherine Lewis, (2002a) that, "'It's not so much what happens in the research lesson itself that makes it successful or unsuccessful. It is what you learned in working with your colleagues on the way there'" (p. 11).

Summary

There are many studies that examine in some way how teachers use, value, and learn about student thinking. Steffe and Thompson (2000) described how teaching is the premier venue for understanding student thinking. Many reports indicate that an examination of student thinking provides teachers pivotal opportunities for change toward reform oriented philosophies (Ambrose, 2004; Empson & Junk, 2004; Simon & Schifter, 1993; Steinburg, Empson, & Carpenter, 2004). However, there are few if any reports of what it means for preservice teachers to anticipate student responses and thinking. An answer to this question enables researchers to design interventions that are able to take advantage of the rich domain of student thinking.

This chapter has reviewed literature on research into student thinking, what teachers know of it, and what teachers learn and gain from it. Many programs have been designed to increase teachers' knowledge and use of student thinking. Lesson study is a form of professional development with several components that offer teachers opportunities to learn about student responses and thinking. ASR has been defined in terms of what it is about (students' conceptual development) and what its purpose is (to enhance the learning experiences of the students). Anticipated student responses stem from and shape lesson tasks and questions. Theories about anticipation and knowledge of student learning indicate that anticipating is natural and useful in predicting outcomes of lessons so that teachers may better prepare to teach. The components of lesson study form a cycle of improvement in both the lesson and the lesson researchers' abilities and knowledge. Goals are very important and lead to the generation of lesson ideas and tasks. ASR is used to refine the lesson, inform researchers of students' conceptual development, and prepare teachers to enhance students' learning experiences. Observation and reflection of the lesson lead to greater knowledge of students' conceptual development, refined teaching abilities, and a better designed lesson.

The purpose of this research is to characterize student responses by preservice mathematics teachers in a lesson study group. What does anticipating student responses mean to preservice teachers, and why do they do it? Hints that ASR is not well understood by teachers new to the lesson study process are briefly noted in several papers. Fernandez et al. (2001) stated, "One group…went through the motions of generating a list of anticipated student responses, as required…however, the group did not take these anticipated responses into account when planning a class discussion" (p. 9). The Fernandez study raises questions for further research about the abilities of preservice teachers, unfamiliar with lesson study, to anticipate student responses in the conceptually oriented ways the experienced lesson researchers do.

Methodology

Case Study Methodology

This research examines how anticipating student responses is characterized by preservice mathematics teachers in a lesson study group. A case study methodology provides direction in conducting this research. A case study methodology is defined by a research question that calls for the examination of a "particular situation, event, program, or phenomenon" (Merriam, 1998, p. 11). Further defining points of case studies is that the researcher has little control on the event, treatment, or participants, and the desired end product is descriptive (Merriam, 1988, p. 9). This study meets these descriptors in that the research question asks how ASR is characterized, there was no control on the participants actions and ideas, and the desired end product is descriptive in nature. Another predominant feature of case studies is that case studies are studies of bounded systems, "that is,… an examination of a specific phenomenon such as a program, [or a] process" (Merriam, 1988, p. 9). This research was conducted in a bounded system in that the participants were enrolled in a particular course during the period of the Fall semester of 2004 and data was collected from one LSG during this semester.

Stake (1994) described two main types of case studies: intrinsic and instrumental. An intrinsic case study is "undertaken because one wants better understanding of [a] particular case" (p. 237). The particular case is the issue of interest, for example, Why do folks from Pratt, Kansas tend to vote republican? Or, What is the diet of latchkey kids that attend the local elementary school? In an instrumental case study, "a particular case is examined to provide insight into an issue or refinement of theory. The case is of secondary interest; it plays a supportive role, facilitating our understanding of something else" (Stake, 1994, p. 237). Examples of instrumental case studies relative to the examples of given for intrinsic case studies would be something like, What economical factors play into the development of the political ideology of residents of Pratt, Kansas? Or, What diet options are readily available to particular latchkey kids from the local elementary school? (p. 237). Hence, this research is an instrumental case study because the case played a supportive role necessary to understanding the particular research question. Other groups could have been chosen and ASR could have been examined under other settings, but the central purpose is to understand developing or initial conceptions of ASR.

Cresswell (1998) further identifies case studies by purposeful sampling, data collection that is extensive combining various forms of data to provide a full picture of the concept of interest, and data analysis that is conducted by interpreting data in context (p. 62, 63). This research used purposeful sampling in the choosing of participants that were observed to have different ideas about teaching and mathematics and were vocal with their ideas. Various forms of data were collected which will be described later as well as the analysis of the data which involved interpreting the discussion of the participants. Also fitting case study methodologies, the researcher had a friendly but unobtrusive relationship with the participants to enable them to freely voice their opinions and ideas without being directed (Cresswell, 1998, p. 117).

Structure of the Course

Research was conducted in the methods class in the fall of 2004 at a large private university. This methods course is a class where Mathematics education students are taught concepts and principles of teaching mathematics. The class was structured so that students taught a 30-minute mathematics lesson to their peers in both the first and second half of the semester. Topics for lesson plans were pre-determined and included units from both traditional and reform textbooks aimed generally at the 7th to 11th grade. Members of the class were assigned to play the role of corresponding grade level students and were specifically instructed to think, but not act like students in those grades. A 15-minute reflection meeting followed each student's lesson where students were given roughly 8 to 10 minutes to ask questions or make comments about the lesson after which the instructor took 5 to 7 minutes to do the same. Students were instructed to direct their attention and comments to the content of the lesson and not on the presentation abilities of the teacher.

Two members of the class were assigned to observe rather than participate in each lesson in a rotation that allowed each student to perform this observation task four times during the semester. The students assigned to observe knew in advance that they were to observe a particular lesson and were also required to write a lesson plan for that lesson. Observers were specifically instructed to find as many different ways students were solving or thinking about the given task as possible.

Each week the students were taught principles and discussed issues in mathematics teaching. Because students would eventually be grouped into lesson study groups and spend the semester operating in that framework, lesson study was a major topic of seven weeks' of instruction, and was frequently referred to and discussed throughout the rest of the semester. Students read "Lesson Study: The Core of Japanese Professional Development" by Lewis (2000), and read most of *The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom* by Stigler and Hiebert (1999). Additionally, the instructor modeled a research lesson that he had observed in Japan. He had observed the creation, teaching, and reflection of this lesson. Other topics taught during the semester included open-ended questions, procedural versus conceptual teaching, teaching philosophies or cultures in other countries, nurturing persistence, instructional tasks and the cognitive level of each task, exam writing, assessment, questioning strategies and discourse concepts. Instruction about anticipating student responses was woven in the basic instruction about lesson study. It was a topic of class discussion in introducing lesson study, writing lesson plans, and reflecting on lessons. To help students understand what it means to anticipate student responses and thinking, a special lesson was given where all the students were to solve the same problem in groups and display it on a poster board. The instructor used these different solutions to highlight the different sense-making inherent in the different solutions and they were instructed that this sense-making was the object of focus of anticipated student responses.

Students enrolled in this course were duly enrolled in an observation class where they made 10 different observations of practicing teachers in the public schools. These observations were based largely on the observations outlined by Artzt and Armour-Thomas (2002) in their book *Becoming a reflective mathematics teacher: A guide for observations and self-assessment.* These observation assignments included the observation of discourse, tasks, lesson phases, learning environment, and monitoring and regulating. The last observation assignment was to observe another lesson study group from their class teach their research lesson in the public schools.

One of the core goals of the methods class was that the students would learn to see mathematics from a conceptually oriented view as opposed to a procedural view. This goal was encouraged as students prepared each lesson plan by first writing what is called the Big Mathematical Idea (BMI), which is a description and explanation of the mathematical concept to be learned through the lesson. It was also hoped that students would learn to specifically observe how mathematics is taught, in other words, what the teacher does to help the students learn and understand mathematical content. Members of the class were instructed that while some things are important to teaching such how a fellow member manages the classroom noise or unruliness, or presents themselves with a quiet or loud voice, charisma or smiles, these things would not be focused on in the class because they were secondary to what really mattered, teaching mathematical content. Focusing on mathematics content instruction was encouraged as the instructor focused his questions and comments in the reflection phase on the content of the lesson and not the presentation. Additionally, the students were instructed roughly halfway through the semester on how Japanese practitioners conduct Hansekai, or “reflection meeting.” As reported previously, lesson study reflection meetings are very focused on how students learn and what can be modified to improve the lesson to enhance the learning opportunities of the students. Members of the class were instructed to plan their lessons using principles and ideas that were taught in the context of lesson study, therefore, the quasi-reflection meeting after each student’s lesson.

The instructor had used lesson study as a framework for the methods course for over four years and structured the course to follow the principles of lesson study within the constraints of the semester. All students wrote a single lesson plan each week in a prescribed format that is similar to lesson study lesson plan formats. There is a column for questions, anticipated student responses, and teachers plan for responding to the anticipated student responses (see Appendix B).

Participants

In the third week of the semester, the 21 students were assigned to groups of 3 or 4 which formed their LSG. Groups were formed to include students with both conceptual and procedural views of mathematics. These beliefs were exposed as students responded to the first journal prompt which consisted of three questions:

1. How do you learn math best?
2. What is mathematics?
3. What do you look for when you observe a mathematics lesson?

Both the principal researcher and the instructor of the methods class read these journal prompts to form a consensus on the view of mathematics and mathematics learning that students held. Some students could not be clearly identified as having either a conceptual or a procedural view. These students were viewed as having mixed or otherwise indiscernible viewpoints. Groups were formed to have students from opposing perspectives relative to their responses to the first journal prompt questions. The assignment given to the LSG's was to construct a lesson that they would teach first to their peers in the middle of the semester and again at the end of the semester to a public school class.

One of the groups was formed specifically for this research with the additional characteristic that in-class observations revealed that they were given to verbalize their thoughts, ideas, and questions. The four participants who consented to be participants in this study are Monty, Amanda, Felicity, and Julie (pseudonyms). Of these four participants only Felicity was not a mathematics education major. She was a history teaching major with a mathematics education minor. Because of her major she was surprised to be asked to participate in this study. She described herself as a person with lists. She often helped the group stay organized by listing at the end of each meeting what they still needed to accomplish. Monty had been a mathematics major but switched to mathematics education the summer before this research began. He worked every other day as a teaching assistant in mathematics classes at a local high school. He described himself as a student who tended to perform at a level that was acceptable to him. More specifically, that he tended to complete assignments to a level where he felt that he understood but did not tend to work toward full expectations of teachers. He was very involved in intramural athletic activities, and his father was a high school mathematics teacher. Julie was in some ways opposite of Monty. She was a perfectionist. She met with professors often to make sure that she was understanding at the level required for the mastery of a subject. In the course of their LSG meetings, Julie had difficulty thinking about lesson ideas when some material they had planned on using was not entirely prepared. She needed the materials to be prepared before she could think about how students would work with them. Amanda was coded as being conceptual. Though she felt that conceptual teaching was more valuable than procedural teaching, she had some question regarding how to do it. She expressed that conceptual teaching appealed to her despite her feeling of inability to do it well. In developing the lesson, Amanda often lobbied to design the tasks so that students would have more ways to solve them, when others suggested closing off lines of mathematical reasoning.

Data Collection

In keeping with the case study research tradition, this study drew upon "multiple sources of information such as observations, interviews, documents, and audiovisual materials" (Cresswell, 1998, p. 62). These forms of data that Cresswell described were collected in this research. Data collection that was native to the course include: videotaped lessons of individuals two times during the semester, videotaped LSG meetings, copies of dialogue journals, copies of individual lesson plans, copies of LSG lesson plans, videotaped research lessons to both peers and later a public class, a copy of the final written report of the LSG, and copies of other written materials. Initial and final individual interviews with the participants comprise the only data collection from the participants extraneous to their course assignments. Additional data was collected through field notes of the instruction given to the students, the classroom experiences of the students, and of the LSG's meetings.

The researcher met weekly with the instructor during the data collection process to discuss findings and observations. These meetings benefited the researcher by gaining insight into the meaning of ASR from the instructor's point of view; also, the instructor occasionally suggested specific observations to make. The instructor received and returned all coursework to the participants then the researcher solicited these returned assignments from the participants. The researcher videotaped all group meetings, attended the methods class, and conducted all the interviews. No attempt to intervene in the lesson study process was made by the researcher. The participants were informed that no specific data would be shared with the instructor until their final grades had been submitted. Every few weeks the researcher suggested journal prompts to the instructor to attain information relative to this research.

Primary Data Sources

Videotaped Individual and Group Lessons and LSG Meetings and Copies of All Lesson Plans. Each participant taught two lessons during the semester. These were all videotaped. All of their written lesson plans were copied and used to both form initial characterizations of ASR and to corroborate with findings of the analysis of the videotaped LSG meetings. The participants met as a LSG for nearly 16 hours. They met six times before teaching to their peers and seven times between teaching to their peers and teaching to the public school class. They also met and were videotaped three times after teaching the public school class to organize the final write-up of the lesson research experience and to participate in a formal in-class reflection meeting. (For a chronology of LSG, meetings the duration of each meeting and a short description of what they did during each see Appendix C).

Initial and Final Interviews. Before the participants began meeting as a LSG an initial interview was conducted with each individual. At this time participants had been instructed on the basic components of lesson study including ASR and two had already taught their first lesson of the semester. The purpose of the initial interview was to provide an initial image of the participants' concept of ASR. This was done with both a problem that participants were asked to solve and then anticipate student responses for and prepared questions (Appendix D). The final interview (Appendix E) was conducted after the LSG had met for the last time and had another problem for participants to anticipate responses for and questions designed to draw out the participants' conception of ASR. Central questions to these interviews were, “When you prepare a question or a task and you anticipate student responses for it, what kinds of things do you think about? What do you think it means to anticipate student responses? What do you get out of anticipating student responses?” The problems in both interviews were chosen because there were different ways to solve them. Additionally, the final interviews were used to prompt participant’s experiences in the LSG relative to ASR, for example, if they felt they should have anticipated responses other than what they had anticipated and how they went about anticipating student responses. Particular questions were prepared for individuals to understand the rationale for some of their group and individual decisions. All interviews were completely transcribed by the researcher.

Field Notes. Field notes were taken in both the methods class and the LSG meetings. Notes taken in the class chronicled participants’ experiences and specific instruction. Attention was given not only to instruction relative to lesson study provided by the instructor, but also to the nature of the teaching experiences in general for all students and specifically for the participants of this research. For example, notes were taken on the nature of the reflection meetings following students’ lessons, and the emphases of the instructor. Field notes taken during the LSG meetings recorded the discussion topics of the group and provided initial insight into the participants’ characterization of ASR.

Secondary Data Sources

Dialogue Journals. Journal prompts were given throughout the semester as the researcher or instructor felt necessary both for the students’ growth, and for this research (See Appendix F). The main purpose of these prompts were to receive a direct written response by the participants to questions about their conception of ASR and its purpose or function.

Final Written Report and Other Written Materials. A report of the lesson study experience from the participants as a group and as individuals was prompted by the instructor. Additionally the groups final lesson plan was copied (Appendix G). Additional written materials include all other class assignments.

Data Analysis

Data analysis of case studies is inductive. Merriam (1988) wrote, “Inductive means that, for the most part, case studies rely on inductive reasoning. Generalizations, concepts or hypotheses emerge from an examination of the data – data grounded in the context itself” (p. 13). Initial analysis began during the data collection process. Field notes and observations, journal prompts, and lesson plans initially provided some orientation to the characterization of ASR by the participants. These initial characterizations were presented to the researcher’s thesis committee and they suggested that a careful examination be made of the LSG video data and interviews to identify the extent of the rationale for their decisions and the depth of mathematical reasoning used to make decisions.

All LSG meetings were then viewed and partially transcribed by the researcher. They were first viewed to outline discussion topics from one topic to the next. Then moments that were noted as containing relevant dialogue to ASR were thoroughly transcribed and the other moments in the dialogue were transcribed to convey the larger context of the conversation. In capturing as much of the dialogue relevant to ASR as possible, a very broad definition was used. Any comment about what their students would do or think or say, or, when they spoke as if they were a student was noted as an anticipated student responses. Dialogue noted as relevant to ASR was most often identified when participants used the pronoun “they,” for example, “what if they don’t know to convert to minutes?” *They* referred to the students they would be teaching.

Some anticipated student responses were less direct, for example, given the tasks the participants initially designed some students would have different amounts of work to do. They anticipated that some groups of students would take longer or have to work harder than others, but they wanted to make the work load as even as possible. This is an example of what would happen to the students given the task, and though it was not a conscious response from the students, it was an action through the students that was anticipated. During preliminary analysis, the unit of analysis was the central idea or ideas contained in student responses anticipated by the preservice teachers during discussions. In other words, what was the LSG talking about when they anticipated student responses? Initial analysis was conducted in effort to answer the question, “What were they anticipating?” These were identified and coded.

Once central ideas of anticipated student responses were identified, a secondary analysis began with the purpose of finding motive or rationale for what the LSG anticipated. For example, anticipating certain responses for the purpose of working out the logistics of the lesson, or, making the main point of the lesson emerge more naturally. The secondary analysis was done by both a careful examination of the transcripts, and a review of the video data. An additional purpose of the secondary analysis was to reevaluate the codings in the first analysis, to search both for alternate or more appropriate codings or otherwise confirm or disconfirm the coding of the first analysis.

The objects that were identified in the first analysis were then organized into groups according to the rationale for each as was reflected in both the transcripts and video data. This was done in order to answer the questions, “Why does the LSG anticipate these things? Of what value are these anticipated student responses to the LSG?” These groupings were then scrutinized through additional review of the video data to determine if they were an original grouping or if they were a subset of some other grouping.

During the writing of the analysis, the video data and transcriptions were reviewed again to find relevant dialogue that highlighted the findings and to check the accuracy of both the findings and the transcriptions. Once the bulk of the findings were written, the interviews were reviewed to find confirming or disconfirming evidence to the findings and provide insight into each member’s particular efforts. Additionally, secondary data sources, such as lesson plans and journal prompts were reviewed at this stage to determine if what was reported in them was different or supported the findings.

Findings

The first half of this chapter will report the development of the lesson before the LSG taught it to their peers. Particular emphasis is given to the major factors that influenced their decisions and an equal emphasis is on the object of the LSG's anticipations. Following a report of the lesson as it was taught to their peers, further revision of the lesson is discussed with particular emphasis on the purpose of the LSG's anticipated student responses.

Meeting With the Cooperating Teacher

The LSG was assigned to teach a lesson in Mrs. Peabody's (a pseudonym) 9th grade class. Mrs. Peabody had four years of teaching experience and was asked to host the LSG because her philosophies and practice were in harmony with reform mathematics. The class of students that the lesson would be taught to was 9th graders in Algebra 2. Her students were accustomed to working on challenging open-ended problems and communicating and justifying their ideas and conclusions.

The LSG met with Mrs. Peabody on September 30, 2004 to determine a lesson goal. She was instructed by the university methods class instructor to think of an overarching goal that should be addressed through the groups' lesson. It was suggested that the overarching goal might come from the process standards of the 2000 NCTM standards which include: problem solving, reasoning and proof, communication, connections, and representation. Her lesson goal was,

> Everything I do I want them to be able to learn to make connections. But the connections I want them to make are between some real… physical situation like people walking at a certain speed or a ball bouncing or

> something like that and a graph. Because I think what they do is they graph and they talk about physical things or even make equations of physical things but they never connect that a graph is a picture of it…. I don't know if you want to come up with different situations and even get different data or whatever or collect data in the classroom, I don't know how you want to do it. But, somehow I want to connect the physical, something's happening and then a graph of it. That's my idea…. They know how to graph lines and they'll know quadratics by then, but even anything else will be fine because they'll learn to graph anything else later on, and it can be introduced then. So whatever works I guess I leave that part up to you.

There were 19 students and most of them were in other advanced classes for their age. When asked about the students' attention span and discipline Mrs. Peabody said:

> There is absolutely no disciplining. They work hard and they can be challenged and they will take the challenge so you might want to make it pretty hard. All of them will do their work and all of them will try and most of them will come up with a solution.

Developing the Lesson

The LSG met six times for a sum of 6 ½ hours between meeting with Mrs. Peabody and teaching to their peers on October 13, 2004, a mere two week time frame. The first two of these six meetings summing to nearly an hour and a half were spent brainstorming to write a BMI in light of the lesson goal they were given. The next three meetings were spent developing and detailing the central lesson idea and tasks and the last meeting before teaching to their peers was spent outlining and talking through the lesson.

Brainstorming and Formal Writing of the BMI

As the LSG began meeting they quickly brainstormed about some plausible lesson ideas but kept coming back to the necessity of writing the BMI. They decided to focus on writing the BMI so that it would guide their decisions about lesson ideas. They expanded on the lesson goal that they were given by Mrs. Peabody by listing every possible connection that they could think of. They listed all of their ideas whether they liked them or not, relative to the idea of connecting graphing with something that is happening. The something could be any physical happening as directed by Mrs. Peabody. Then they sifted through their ideas for the BMI deciding which phrases to keep and which to discard based on whether the group thought the idea was central enough or if they would have time to do it. Eventually they all concurred with Amanda that, "Seeing the big picture and interpreting what's going on," was of primary importance. Ultimately they each wrote a possible BMI and combined the language of the BMI's they liked the best to form the BMI for the lesson. It read as follows, "Your life has mathematical data that can be graphed. These graphs show us that something is happening. We can use direction and slope of graphs to interpret what is going on. We can also use these graphs to predict similar future events."

The LSG believed that *interpreting* combined many of their ideas and it was used to imply the mathematical ideas of collecting and organizing data, graphing, direction, predicting, and understanding what is happening at a point or moment. These ideas were all very important to them and were implied in the BMI even though they were not explicitly stated. The BMI brings to mind a crude outline of a statistical query where data is collected, represented, and analyzed. Indeed, this formed the basis for their lesson design. Note that the BMI does not prescribe any particular lesson and that it is a sort of hybrid between a lesson goal and a mathematical idea. The first two sentences are quite general and serve to meet the lesson goal given to them by Mrs. Peabody, specifically that students make connections between real life and mathematics. The second two sentences are more specific and lead the LSG to a particular mathematical emphasis, namely interpreting the slope of a graph as speed in the context of the lesson they design. The last sentence in the BMI was eventually dropped because they felt that they did not have enough time to address the issue.

First Shaping of the Lesson Tasks

There are countless different lessons that could be prepared to meet the BMI. The LSG came up with an original idea for the lesson, but it did not come to them easily. A couple of them initially looked for lesson ideas on the internet and other places, but none seemed to fit so they made up a lesson from scratch. There were a great many ideas that had to be generated and details that had to be decided on. It will be shown that the major tasks of their lesson were under construction right up until they taught it to the 9th grade class.

In the group meeting of October 7, their first meeting after writing the BMI, they spent 110 exhausting minutes discussing and thinking through ideas before their lesson idea had some form and direction. They decided that the students would be put in groups and would be taking a hypothetical car trip from a common starting point to a common destination. Different routes were possible, and each group followed a different route.

Each group would receive a table of data (for an example see Table 2) that consisted of the distance and velocity of each segment along the route. The first task was to solve the table for the time necessary to complete each segment. Once groups had completed the table by solving for time, they would be directed to make a distance versus time graph (see Figure 1) using the accumulating distance and time from their tables. In the second task, with all of the graphs displayed before the class for inspection, students would be given the driving directions of all the routes and would be asked to coordinate the graphs with the routes. Labels of the graphs and routes would be masked to prevent students from immediately knowing which route matched with which graph. In order for students to match the graphs with the corresponding routes, they would have to figure how to coordinate the driving directions with a particular graph. Finally, they planned to have a class discussion on how students had matched the graphs with the routes.

Table 2

Data for a particular route.

Distance	Speed	Time	Total Distance	Total Time
2	20	6	2	6
0	0	1	2	7
5	25	12	7	19
0	0	1	7	20
2	35	3	9	23
0	0	1	9	24
3	25	7	12	31

Note. Gray cells were to be filled in by students.

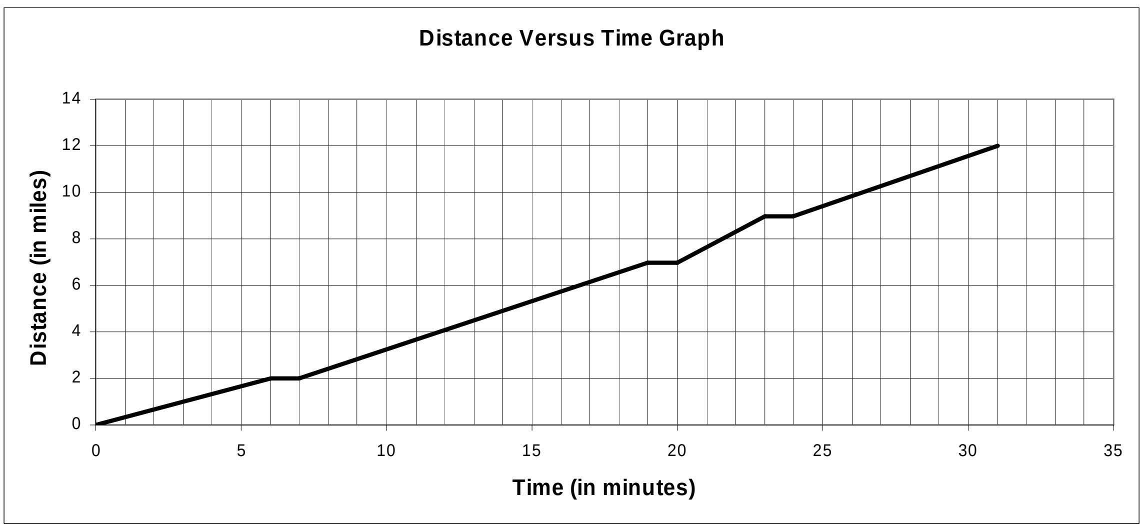

Figure 1. Graph of accumulating distance and time from Table 2.

Defining and Refining Initial Task Parameters

Anticipated student responses was often a part of their discussion as they planned the lesson and their anticipated student responses stimulated changes in the lesson. The following excerpt taken from the first few minutes of the group meeting on October 7, 2004, demonstrates that ASR was something the group did naturally in order to shape and define the task. In the following quote, Felicity has a question about the particulars of the routes they will create. She feels that there may be a problem, but is not sure. Julie makes a decision that the group accepts without comment and without discussion on whether there really is a problem with the issue.

> **Felicity:** See the one thing that I'm already wondering? Maybe when we try it, it will work out. If we have somebody go [on highway route] 265 and tell them they can go 50 miles per hour and tell them how far it is with three stop lights, it will take them five minutes and so the other group that's going to have to go farther, is that going to cause a problem? See, 'cause this group will get there in five minutes and then another group goes down and takes [highway route] 89 and it takes 10 minutes. Does it matter then?
>
> **Julie:** Lets make it so it takes about the same amount of time. Like we could plan it out. Well, then we would be giving them the data. Well ok, what if we gave them the table, gave them the map and gave them the table?

Felicity identified this issue of making the routes similar by anticipating how the lesson would work out in the classroom and sensing that there may be something problematic with some routes being faster than other routes. The issue of route homogeneity would eventually include many other parameters. They continued to define and limit the parameters of the routes as they anticipated how students would be using and thinking about them. Later in the same meeting, Felicity returned to the concept of making the routes similar and explained another way in which she felt they ought to be similar.

Felicity: See the freeway works, except you cannot allow a group to just be on the freeway. Because then if it's like 65 mph…

Amanda: Well, they'll have a road to get there, and then roads to get off.

Felicity: Then we need to have construction or something because I'm worried… you have group one over here with their freeway and they're like, "Point, point, point. We're done." And, this other group here is like, "A point, a point, a point, a point, a point, a point." And they're going to be taking a lot longer. Do you know what I mean? Like, we need to have something else so that to some degree it comes out to be equal, just in the work they are doing. You know what I'm saying?

Amanda: Yeah, I hear what you're saying, but I'm just trying to think. See we need someone on the freeway so the graphs end up looking a lot different. But we also, I just don't, …I'm afraid we'll have this group one…

Amanda: Maybe on the freeway we don't give them the actual miles. We make them figure it out. Do you think that's mean?

Felicity: Like that one group that gets a ruler in their packet. (Both laugh.)

The group was able to identify and normalize those parameters of the task that could be problematic for the students in some way by anticipating student responses. Their goal here was to make the make the amount of work that students would do and the complexity of their work be equitable. They identified the potential problem of inequity by running through the lesson or lesson idea from the students' perspective. In this way, the group naturally anticipated student responses and thinking to modify their task ideas.

In other words, they anticipated how the tasks would go and in doing this, they identified weaknesses in their tasks' design.

Associated with the details of the routes was the major issue of how students would retrieve the information from *their* route to complete the table. The LSG decided to employ maps. Using maps allowed them to make the lesson a little more true to life for the students, since the map would be of the students' area and they could have speeds of segments that were truthful. An idea they discarded quite early with the maps was that students could use rulers and the scale on the map to determine the length of different segments along their route. It was decided that it would take too much time for the students to have to determine the distances of segments this way and that the students might not be very accurate. Instead, they decided to color code the segments according to velocity so that students would still use the maps and then give the distances of each segment on a handout. They felt that this idea was both faster and more accurate for the students to use.

Making the lesson realistic to the students was very important to the LSG; it was part of the goal that Mrs. Peabody had given them and was implicit in their BMI. Yet they had to compromise a little. By generating the idea to color-code segments of each route, where each color would indicate a particular speed that would be placed in a key on the map, students would not very realistically be collecting data.

> **Felicity:** So you think, you don't want to make up data basically. What I'm wondering is we're trying to use the Provo maps so it really is real life for them like, "Wow, you would actually be doing this someday." I'm just wondering we could take out a lot of the real difficulties. We could just

say, we could really check and this is University Parkway and you can go 45 mph on University Parkway, and they actually are driving from here to there. There actually are 7 lights. And so we could go, "It actually is 45 mph. There are 7 lights." or we could go, "On this road we want you to go 40 mph." It would kind of take away the… reality that this is Provo. We could go, "On red colored roads you go 45 mph, and on black colored roads you go 30."

Amanda: That's a good idea.

Felicity: And so that way it's not completely real life but we can take control of the map. And if we are afraid they are going to question us, that that is not really how it is, we could make up a completely different one, you are going to go from here, and it could be a real place but it's not like here in Provo.

In the excerpt above the LSG anticipated the feeling that students would see their tasks as true to life. The LSG wants the stop lights on the map in the task to be in the same place they actually are in the city and the speed limit on roads to be accurate. However, they anticipated that having things true to life, though appealing to both them and their students, would not allow them to make the routes meet their ideas about being similar. The group began to compromise a little on the true to life aspect of the lesson. The true to life aspects were messy for the participants and would take too much time for the students to work through. Additionally, the preparation of this map will prove to be tedious because they would need to carefully color-code segments of routes on as many maps as there are students.

Helping Students Through the Mathematics

In order for students to collect the information of each segment of their route, they would have to calculate the time it would take to travel along each segment given the particular velocity and the distance. This was another way in which the LSG felt that the lesson would be true to life in the sense that the students were collecting or recording data. The problem students would face at this point was how to take distance and speed and find the time. The LSG had written the formula "$D = S \bullet T$" for the *distance* equals *speed* times *time* ($d = rt$), on the whiteboard in the study room they were using. When they decided to give students distance in miles and speed in miles per hour, (having considered other possibilities such as having students do a speed versus time graph), they used the formula to work the problem out for themselves. They had created a table of information that represented the distance and speed of a possible route, and then used the formula to solve for time. Felicity anticipated that filling out the table would be "challenging" for the students, probably because when she filled out the table with Julie and Amanda it took some thinking to solve for time. They did not discuss at this point the challenge that it would pose to students, but seemed to anticipate from this experience that students would use or would have to use the formula $d = rt$ to solve the table for time, and they were aware of the necessity to convert the unit from hours to minutes.

In the following meeting Monty, who was absent from the previous meeting, anticipated that students might have difficulty solving the table for time in minutes. He asked, "Are we going to run into any problems with giving them 35 mph and we are working in minutes?" Felicity and Julie did not respond directly but took information from a route they had created and solved for time, again using the formula $d = rt$. They did not discuss the difficulty of solving for time at this point as Monty was asking about, but turned their attention to generating helps that they anticipated might give the students enough information to successfully complete the task. In the previous meeting they thought that they would include the formula $d = rt$, but here they decided against it. Part of the reason is at least indicated by their confidence in the students. They were informed that they would be teaching bright students, so they anticipated the students being able to solve the problem without any help.

> **Felicity:** I don't want to tell them that there are 60 minutes in an hour because that's obviously stupid. But I feel like I need to give them a hint, to multiply everything times 60. We're giving them the hint about *distance equals speed times time*. I'm wondering if we should give them the hint already or if we should let them figure it out themselves.
>
> **Amanda:** I think we should let them figure it out.
>
> **Monty:** I think these kids can figure out even the equation *distance times whatever*.

They anticipated that the students were likely to figure things out, but they also hypothesized that there would be some difficulty, and while they knew where the difficulty lay, they named it rather than getting at the source. It seems like they named a symptom, but did not identify the cause of the malady. As they walked through the lesson immediately prior to teaching it to their peers the issue came up again. They had already expressed confidence in the public students being able to successfully fill out the table. Nevertheless, they worried about the students knowing how to convert the time to minutes.

Julie had been chosen the previous day by random drawing to be the teacher. She talked through the lesson with the other group members acting as students and researchers. They did not plan to give the formula $d = rt$, and they knew that by using the formula to solve for time, given distance in miles and speed in miles per hour that students would necessarily have to convert their immediate solution in hours, to minutes. Most of their surrounding discussion was on the lesson or task development with questions to help overcome the difficulty and create a smooth running task. Absent was a discussion on why students would have this particular difficulty, which is peculiar in light of the instruction they received relative to anticipating student responses. Though they explicitly stated that the difficulty was converting to minutes, they did not explicitly say why this would be difficult. Their discussion to develop helps implied that one of the reasons for the difficulty was that the students may not be aware that they have to convert to minutes, or, that they may not realize that their calculation of distance divided by rate would give an hour unit, but they did not specifically spell them out.

Monty: You might want to give a hint. To convert time from hours to seconds somehow.

Julie: Ok

Amanda: I think when you are walking around and say, "What does that mean? And see if they can get it on their own.

Julie: And maybe stand a minute at each table and see how they're doing ….and then if they needed it give them the equation or say, "How do you change it to minutes?"

Amanda: Just say, "What does this number represent? What does it mean?" Then they'll think about it for a minute, and then ask them what time frame.

Felicity: If they figure out *distance equals the speed times time* and they don't know to multiply it by 60, they're going to get numbers like point oh (0.0).

Julie: That's true.

Felicity: That would be a good time to use the hint, "What does this number mean?" …"Could you go 6 miles in 1.2 something?" Something like that.

Julie: Ok… So then we'll talk about it. "What are some struggles that you ran into when you tried to do this?"

Monty: Good question.

Amanda: "Figuring out the time in minutes."

Julie: Ok, how did you finally do that?

Amanda: …"We realized it was in minutes. We wanted it to be in minutes so we had to multiply by 60."

Amanda: Maybe with the time we should say if we want it in minutes, hours.

Julie: Should we?

Amanda: Cause we put all of our graphs in minutes.

Julie: In [the 9th grade] class if we told them first that they were going to graph it then they would know that…

Felicity: Maybe you could just wait and see what everyone is doing as you walk around the tables. And then if one table has multiplied by 60 and they're getting real things, then you could like ask them… because they could fill out their entire table I guess that way. And you can ask them what they did differently and then ask the class what makes more sense.

Monty: Isn't the BMI to try and make a connection between a graph and what we are doing? So is that necessary to have them struggle with minutes and hours and stuff to accomplish our BMI?

Julie: No I don't so much want them to struggle with minutes and hours and seconds but…

Monty: So you might want to direct them that way because that's not something we're going for…

Julie: So someone might come up with the equation but if not, well I was going to have someone like show or talk about how they did it. And then, ask if anyone did it differently. And then ask if anyone, did… use an equation. So, if they did then they say it and I put it up. Or, if not should I be like, "If I gave you the equation would that make it easier?" Or what? How should I if they don't bring it up.

Monty: I think today that will happen… I really liked your question earlier when you said, "What were the problems you ran into?" I think when you ask that, the class, then you can give them a little more time after people brought up what they ran into. Cuz when you talk about the problems you run into, minds start working and thinking, "Oh wo that was a problem, oh this is how you solve it." And that will come up, I think. I think today that will come up. I don't think you need to say... You know what I mean? I'm pretty sure that will come up just because we don't know how to think what we need to think like….

Julie: ….Then probably give them a few more minutes to fix their tables if they need that.

This excerpt above highlights two important factors in the planning of this lesson: anticipating difficulties, and changing their focus to avoid difficulties. First, as the LSG anticipated how the lesson would work out, they anticipated the difficulty of converting time in hours to minutes in the context of planning the lesson and not necessarily toward students' conceptual development. Though it should be said that they anticipated some of the student cognitions that would lead to understanding, yet they did not try to connect how students would come to that with what they would plan into their lesson. Many of the pieces of anticipated student responses in their discussions were not used in the way the literature describes to get at the heart of how the students' understanding would develop through the lesson, and how the lesson would be modified to enhance the students' understanding. For example, Felicity suggested helping the students by asking, "What does this number mean?" And subsequently, Amanda, speaking as a student said, "We realized it was in minutes. We wanted it to be in minutes so we had to multiply by 60." But they didn't discuss how the students would come to this understanding. Their discussion did not explicitly center on why students would have this difficulty. Their discussion centered on getting students to task completion. By the end of their discussion they had ways of getting students to the correct answers but they had not yet fully addressed why students would have difficulty in the first place. This is reflective of the nature of ASR through the discussions of this lesson study group.

Felicity commented that some group might figure out how to convert to minutes and the teacher could use that group to help the others to understand what to do. This type of idea that they could use students to further the lesson because someone will likely get it, occurs infrequently through their meetings. Simon (1995) claimed that this is common among new teachers trying to get students to build concepts themselves. He wrote,

> Novice teachers, who want their students to 'construct' a particular idea, often ask for the idea from the students, consciously or unconsciously hoping that at least one student will be able to explain it to the others (Simon, 1991). Such an approach does not deal with a key question: If a group of students do not have a particular concept, how does a teacher work with them to foster their development of that concept? (p. 140)

The second important factor that influenced the LSG planning this lesson was how their focus shifted as they attempted to reduce difficulties for them and their students. Monty pointed out in the middle of the previous excerpt, and the group seemed to concur by their lack of response, that whatever struggle the students might have, filling out the table was not of primary importance. In anticipating toward designing a lesson to meet the BMI, they did not feel that filling out the table was a necessity. In the final interview, several group members concurred that the purpose of the table was to show where the real data came from, and not so much for students to struggle with. Though, it should be said that not all of them felt this way. For the participants, the primary goal of the lesson was not on filling out the table anyway, it had shifted and continued to shift toward students coming to an understanding that slope represents speed. In other words, the participants devalued the significant mathematics of solving for time in favor of streamlining the lesson towards a different and very specific connection that students would understand that the slope of a distance versus time graph represents speed.

Anticipating Logistical Issues

The LSG had only 30 minutes to teach the lesson to their peers. They knew that they could take an entire hour to teach the lesson to the 9th grade class. Because of this time constraint, the group decided that when they taught the lesson to their peers they would omit the part of the lesson where the students make their own graphs. Instead, they would have an overhead with all the routes graphed. They planned to have students make their own graphs when they taught the lesson to the 9th grade class.

Two of the major issues that they anticipated being problematic with the graphs of the routes, were first, how students would have access to examine the graphs, secondly, how to label them and the routes. Getting students access to the graphs so that they could examine all the routes on the same graph was another difficulty because if the students were to analyze the graphs of the routes carefully enough to correlate them with maps or driving directions, then the graphs had to be clear enough to distinguish one route from another. The LSG was not sure that having a graph of all the routes on the same overhead would clearly distinguish one route from another. Furthermore, in the first task where students had to find time for their routes, they would be given maps that only had their route on it. In order to match graphs with routes, the students would either have to have a map with all the routes colored to indicate speeds, or driving directions for each route. Preparing maps with all routes, where all the routes were color-coded according to the speed on each segment would be a rather long and tedious undertaking. They decided to create two fully color-coded maps per group and they would only have four groups. They would also include the driving directions of each route, which would include distance and rate.

Labeling is a tricky issue for the LSG because they have to label the separate routes on the graph so that students can refer to and identify them, and so that they do not correspond with labels of routes on other materials in such a way that their identity is trivially given away. This issue is a source of difficulty throughout their time together. It seems that they have difficulty keeping track of it in their own minds, whether students will somehow be given so much information about all the routes that matching them with their respective graphs becomes trivialized. In the following excerpt from the group meeting on October 11, 2004, they anticipate this difficulty and also consider if it should be part of the design by considering if it is addressed by the BMI.

> **Felicity:** If I'm in group 4 and I make my graph I would know that graph. Then I would have to go, "Another group is on a slower road." Is this going to give away too much? Is it going to be very very simple to figure out which graph is which? Or, is it still going to require analyzation?
>
> **Monty:** Isn't that our goal? Isn't that the connection we are trying to get them to make?
>
> **Felicity:** I couldn't connect in my head if that was giving them too much information. I guess we don't care if they make a connection between the map and the graph. The connection that we want them to make is with the action and the graph.

Felicity said that the routes on the map and the graphed routes can correlate in label, but the labels on the driving directions cannot correlate with the labels of the routes on the graph because it would make their last task to match them meaningless. Furthermore, there was some confusion as to the focus of the lesson and the specific connections they wanted to make. The reference to the BMI helps them form a consensus on what was important to make happen and allowed them to restrict the lesson in ways that made it manageable to them. Conversely, in identifying what was most important allowed them to manage parameters and issues of the lesson that were not important in ways that were convenient to them.

Focusing the BMI

Throughout the LSG's meetings the BMI was often referred to when making decisions about both the design of the tasks and the lesson plan overall. During the rough shaping of the lesson and tasks the BMI appeared to be sufficiently written to provide direction to the group. But, as they started to work out the fine details of the lesson they felt the need to be more specific. They were anticipating student responses relative to the lesson or tasks in order to determine if the lesson or tasks were in harmony with the BMI. The third sentence of their BMI read, "We can use direction and slope of graphs to interpret what is going on." This provided them with some direction but they needed more. Monty, who was absent on the meeting of October 7, 2004 when the idea of the lesson was created, wondered what they were really trying to teach, and what specific connections they were hoping to make. In the following excerpt taken from the LSG meeting on October 12, 2004 the group discussed possible specific connections they might be after. Amanda was absent at this point in this meeting and the other three expressed the feeling that they needed a more specific goal. They discussed having the goal that students generate the $d = rt$ formula, but discarded it. All three expressed the idea of interpreting slope as speed.

> **Monty:** Well, I don't know what connection we're trying to make. I don't know the specific [connection].
>
> **Julie:** Just that something's happening, like they are traveling and you can see it. That's what I've been thinking.
>
> **Monty:** …. something's happening I think is gonna happen no matter what. I also wrote down the question, "What's the connection between an equation of the line and speed?" I'm just trying to think of connections, specific connections of math and the real world.
>
> ….
>
> **Julie:** [The graph] shows the relationship between two variables, time and distance, and the relationship is speed. Which is what the slope shows right?
>
> ….
>
> **Felicity:** I think it's… I think it's important that we ask questions leading to slope because that's part of our BMI is we say we can analyze [the graph] with slope. So if we're having them analyze them, they're just like, "[the green route] is steeper." You know that's not getting, we need to actually get slope out of them and so they actually get to that.
>
> **Monty:** Right.

Remember that their BMI indicates both a lesson goal to help students see connections between physical happenings and mathematics, and the mathematical concept of interpreting graphs. They have chosen to help students make connections between physical happenings and interpreting representative graphs of those happenings, but this goal is still quite general. They have chosen a physical happening to connect with a graph, but what thinking do they want students to engage in to interpret and understand slope as velocity? The specificity of these is not like that suggested in the lesson study literature, and since their BMI is a bit too general they struggled with what specific mathematics content to teach.

Type of Mathematical Reasoning the LSG Anticipates

Narrowing the scope and direction of the BMI leads the group to anticipate whether the lesson is designed to enable students to make the connection that slope represents the speed on a distance versus time graph. In some ways ASR informed the LSG of the general lines of mathematical reasoning that students could use. However, it is unclear whether these conversations are also used to understand their students' potential conceptual development. Further group discussions suggest that this is not the case. From group meeting dated October 12, 2004.

> **Felicity:** What are some of the questions that we want to ask?
>
> **Monty:**…. "What is the connection between slope and speed?
>
> ….
>
> **Felicity:** If I can see them just going, "Well distance equals speed times time." So could they answer, "Slope equals speed because…" …. I can see them just going, "What's happening here? Well, we've got the equation

distance equals speed times time, since [we have] distance and time obviously this must be speed." I mean we can go somewhere from that. But I'm wondering like to anticipate a response from that.

Monty: I don't understand what you are saying.

Felicity: …. So I can see myself as the student being like, "Why is this valid? Well because my equation already has distance and time. And speed is the other in there, so obviously that has to be what it is." I think we can go somewhere with that but I can still see students answering that way.

….

Felicity: Monty, can I ask you a question? (Pointing to a question written on the whiteboard.) To me this is one of the most important questions "Why is slope equal to speed?" that one. As the teacher, how would you answer that? … If you were the student and you were like the genius of the class, you do have the right answer, what are you saying?

….

Monty: Why is slope equal to speed? Why is it valid?

Felicity: I can see students saying, "On this graph or on this map I can see that I'm going 45 mph that it's steeper…"

Monty: … They'll look at their graph. And figure out the slope of the lines… and they're gonna think, "What is the connection between slope and speed?" And then they'll think some more. And, I want them to be able to say that slope tells us how fast they're going.

Felicity: Ok I can see what you are saying.

Monty: Because of our equation of distance equals the speed times time, or, speed equals distance divided by time... So they'll have… Yeah that's what slope is if we think about it. (Monty gets a graph of one of the routes and finds the slope of the first segment. He talks through his reasoning as if he were a student.) …. They're going to go, "What's the slope between right here?" … In their head or on paper or whatever method they need to do to find the slope, let's pretend it's 8. So *y*'s subtract each other, so distance, so we're going to go 8 minus 13, so five on top…Now the *x*'s subtract each other, 8 minus that. So they're doing an equation in their head so they're figuring out that that is speed…. But they're actually doing the equation and that's what we need to make the connection between.

…. This equation is good for many things and making the connection between slope and speed and this actually finding *m* in our algebra 2 class, subtract the *y*'s over the *x*'s is actually applicable in real life. This graph will show us the change in things. That's what I'm trying to get at it but I hadn't really thought about it until you asked me so that is good.

….

Felicity: To me … a lot of these [questions] are getting to that, the idea that [the graph] is a relationship between the two variables. And that's what our whole slope shows us is what that relationship is.

This excerpt indicates that the LSG used ASR to decide if the students would be able to successfully navigate through the question or task to the desired end and if the end was attainable in terms of what students would be given. The type of mathematical reasoning they anticipated suggests that they had some thoughts about student conceptual development, but they did not discuss how they would use that thinking in the lesson or how they would promote it. Monty's statement that, "I want them to be able to say that slope tells us how fast they're going" indicates that they were trying to funnel the students toward this idea that slope represents speed. Then when he talked through the reasoning of how they could come to this conclusion, he sounded as if he was realizing a way to do it himself and not necessarily anticipating how students would do it. His realization may be conceptual development for himself and he may have thought that students would come to understand in the same manner as himself. Much of this discussion felt like the participants were making sense of it for themselves even though they were speaking as if they were students. There was also an implied feeling that students would come to understand the same way as they had. It is impossible to tell if this is the case, especially since they begin by saying, "they're going to think…." Final interviews and journal prompts reveal that when they anticipated student responses they often thought of how they would understand or do things, therefore, it is possible that while they are speaking as if they are students, they are really thinking of themselves as the students.

Summary and Discussion Ideas Contained in ASR

The preceding discussion and examples represent the most frequently cited objects or ideas anticipated by the LSG. They were anticipating how the task would work and whether students could successfully navigate through it. While they anticipated how the students might understand some things, they did not discuss how students would come to that understanding, nor did they modify their lesson because of how they anticipated students would understand the concepts. These anticipations seemed to the researcher to give the participants ideas of what the students might do, and helped them to feel that the lesson would be successful. In the process of designing the lesson, they first wrote a BMI which also served as a lesson goal. When the BMI was not specific enough for them, they more tightly focused it on the concept that in a distance versus time graph the slope of the line represents velocity. They anticipated student responses related to task design. In summary, the things they anticipated were:

1. That there would be some difficulties or inequities if the routes were not similar. The purpose of their decisions relative to this idea is to homogenize the length in miles and total time of each route, how long it would take students to solve for time, and the difficulty of solving for time for each route.
2. Since they wanted to create a lesson that was realistic to the students, they anticipated realism issues. Realism issues that surfaced were appropriate and realistic speeds for each segment of each route, a map of an area that the students were familiar with, and the process of first gathering data to examine.
3. Whether students would be able to see the graphs of the routes well enough to distinguish them. This is a particular issue of access that the LSG returned to when discussing logistical parameters.
4. They needed to label the graphs of the routes so that they did not correspond with the labels of the routes on the maps or driving directions.

5. They anticipated some of the difficulties that students might have, namely converting time from hours to minutes, and anticipated helps for the students.
6. General lines of mathematical reasoning, with special emphasis on anticipating whether the lesson was designed so that students could successfully meet the goal that slope represents speed.

In each of these ways, the LSG designed the tasks, planned or focused the lesson, and prepared the materials by anticipating student responses.

The participants first formed a lesson goal and BMI, then created tasks and materials in a lesson plan, and then thought a bit more thoroughly about the mathematical reasoning involved. It seems appropriate to outline their work with a diagram (see Figure 2). Sometimes they started by working on the tasks or materials, then considered the mathematical reasoning and the lesson goal, then returned to refine the task or materials. Depending on their purposes, they were seen to start at any location in the diagram and work in either direction and end at any location. The only route that was not observed was the participants thinking about their lesson goals and then going directly to the mathematical reasoning that would be involved. While the literature on lesson study suggests that experienced lesson researchers were able to go directly from their lesson or goals to the mathematical reasoning, the participants in this study did not. This is probably because their lesson goals were not thoroughly defined as they could have been; had they been well defined they may have been able to think about how students would come to understand the concepts in the BMI. Additionally, the participants, being preservice teachers, did not have a great deal of experience to draw upon, and they were new to mathematics teaching and in particular lesson study and had very little if any previous experience thinking about student's conceptual development.

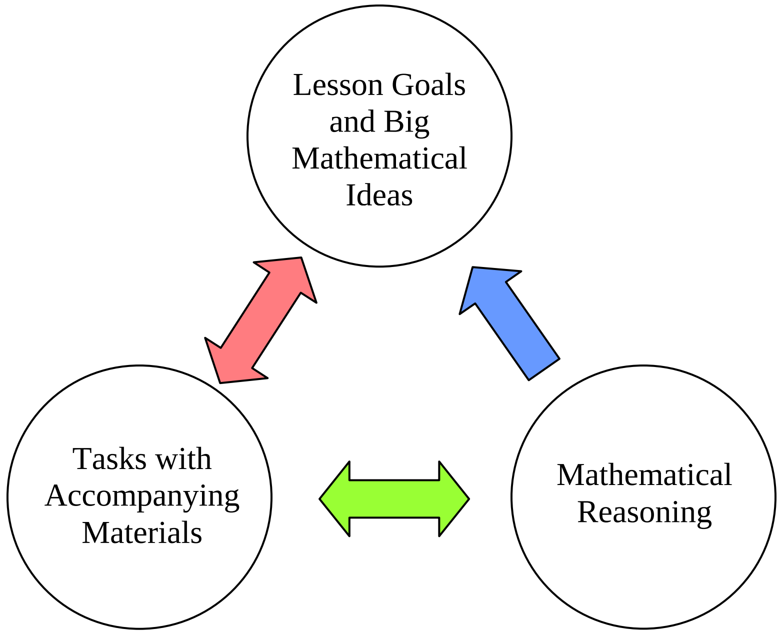

Figure 2. Task development cycle.

Preparing this lesson may have required more work than many lessons because most lessons are prepared where all of the students can work on the same problem or set of problems. In this lesson they had to carefully plan different routes from the same location to the same destination in such a way that they were all graphically similar, equally difficult to solve, but also different enough to distinguish. Anticipating student responses led them to define these task parameters, but once task ideas were set the materials needed to be prepared. Custom crafting of materials was no small task. At different times the LSG expressed anxiety for lack of time to get everything ready. Some ideas began to fall out of favor for this reason. For example, they wanted to have a color-coded map for everyone, but chose to have two per group. It is likely that the number of details to be worked through, and the preparation of materials is part of the reason why they anticipated student responses relative to task design. They didn't have time in their meetings to seriously discuss how their lesson might be modified to help students develop understanding.

A final comment about how they went about anticipating student responses is that it was a very natural, almost implicit part of their discussions. They did not anticipate student responses by saying, "Let's anticipate student responses." Rather, interwoven in their discussions were phrases like, "Is it going to be a problem that…?" Or, "If we give the students *this*, then they will…." Or, "What if they think…?" Notice that the "phrases" were "speculative questions" (Walter, 2004).This natural anticipation was suggested in the literature, and seemed inadvertently reported in literature on lesson study. Fernandez and Yoshida (2004) wrote 200 pages describing the discussions of Japanese lesson study researchers as they prepared and taught one lesson. The discussion reported resonates with the discussions of the participants of this study. Indeed, the anticipating component was so natural that Fernandez and Yoshida spend most of their time discussing the implications and reasoning behind the anticipated student responses that drove the decision making process of those they observed. At the end of their book they only implicitly indicated that all of the decisions were discussed through ASR.

How Teaching to Their Peers Affected Their Lesson

In teaching the lesson to their peers, several important issues arose that would direct the efforts of the LSG as they worked to refine the lesson to teach to the 9^{th} grade class. The first of these was that it took their peers much longer to figure time on the tables. Secondly, students were able to match graphs and routes in ways that the LSG felt did not reach their lesson objective. Resolving these two issues occupied the LSG's remaining meetings.

In filling out the table, their peers used a variety of strategies that they had not anticipated, including proportional reasoning and a type of common sense reasoning. The common sense reasoning was in respect to one of the routes that they had designed to have 30 mph as the only speed. Students were able to reason that at 30 mph, one mile would take 2 minutes, and so filling out the table for that group became quite simple. The experience that their peers had filling out the table led the LSG to make two fundamental changes; the first was to make all routes have a variety of speeds to solve so that the difficulty was even among all routes; the second change was to adjust the timing of the lesson. Knowing that filling out the table would take longer than initially anticipated challenged the LSG to include all the ideas that they wanted to include in the full lesson to the 9th grade class. This led to some streamlining of the task, and the map was ultimately relegated to a role of providing context, but not information.

The second critical issue that the LSG would spend the remainder of their working time to address was that their peers were able to match routes with their corresponding graphs in ways that they felt were unacceptable. Since the students had the driving directions for all routes, they were able to distinguish routes by seeing how far along the distance axis the first segment went and matching that with the route that had gone that distance in its first segment. The routes had a different number of stops, so another way to distinguish their corresponding graphs was to identify the number of stops, and or their location. Another way that the students were able to match the routes with their graphs was to identify the number of segments along the routes. Since each route had a different number of segments, they were easily identified. These methods were unacceptable to the LSG because students did not have to compare the slopes of the different graphs, which was contrary to the goal of the lesson.

Further Refining of the Lesson and the Purpose of ASR

The LSG met seven times between teaching to their peers and teaching to the 9th grade class spending five hours and twenty minutes together. The first four of these seven meetings were spent reflecting on the lesson to their peers and making changes to details of the lesson. The final three of these seven meetings were spent in meeting with Mrs. Peabody to discuss what they had and to ask her some questions, writing up the final lesson plan, and preparing the materials. Anticipated student responses was used in different ways to prepare the materials and design the tasks. Between teaching the lesson to their peers and teaching the lesson to the 9th grade class, the LSG spent a great deal of their time working on details so that students would have to think about the slope of the lines. Instead of giving a list of all routes with driving directions, they gave a list of all routes where the information about each route only included the sequence of speeds.

Having uncovered the ideas contained in the LSG's anticipated student responses, the secondary analysis examined the data to further characterize ASR. In particular, in what ways were their ASR being used to define and refine the lesson? Answering this question will prove valuable because the answer provides a lens through which to examine the creation and evolution of lessons in general, also, this will yield insight into a purpose of ASR and describe how it was used as a tool in refining the lesson.

Five different faces of ASR were manifested in the data in reference to preparing the lesson and creating the tasks. These faces are: (a) anticipating toward the lesson goal or BMI; (b) anticipating the mathematical reasoning that students could use; (c) anticipating whether materials, resources, information or lesson parts were necessary or sufficient; (d) anticipating whether students will be able to work successfully toward the BMI; and (e) anticipating emotional responses.

The First Three Faces of ASR

The first three faces will be addressed together because they were often seen operating in tandem. As the LSG continued to refine their lesson, they filtered their lesson through (a) the BMI, (b) the mathematical reasoning, and (c) the materials or resources. In particular, the LSG anticipated toward the BMI through the mathematical reasoning that students could employ through the materials, resources, and information they planned to give the students (see Figure 2).

At this point of the planning process, the BMI had not been formally rewritten, but they had decided to drop the last sentence about "interpreting similar future events." Furthermore, they had come to interpret the BMI as having the specific unwritten goal to get students to recognize that the slope of a line in this case represented speed, although the BMI still read, "Your life has mathematical data that can be graphed. These graphs show us that something is happening. We can use direction and slope of graphs to interpret what is going on." In refining the lesson to reach the goal that students would connect the slope of a line with speed, the LSG refined the routes extensively and corresponding materials so that the lines of mathematical reasoning that their peers used were shut off. Students would only be able to correlate routes with graphs by careful examination of the slopes on the graphs. They made all the routes have the same number of segments, have stops in the same places, travel a similar distance, and take a similar length of time. Additionally the students would not receive detailed information about each route; they would receive only the velocities of each route in the order they were encountered. In the following excerpt, Felicity initiated the discussion by talking about a lesson goal. She is followed by Julie who then continued the discussion by first describing a task option, that of giving students only the speed. She then describes the mathematical reasoning that she anticipated would come from only having the speed. Felicity followed up by repeating the task option of only giving speed and explaining how it gets students to the goal faster. Referring back to Figure 2, they began at the top with the lesson goal, then moved through the bottom left of tasks and materials, then went to the bottom right which is the mathematical reasoning, then reversed course and returned to the lesson goal.

> **Felicity:** I think the idea we are trying to get across is slope… I think we want them to be analyzing we want them to sit down and do the math themselves.
>
> **Julie:** If we only give them speed what they do is look at the map, calculate the points, calculate the slope, and that's the speed that they have.
>
> **Felicity:** I think just giving them the speeds gets to our BMI faster. Because the speed is what we're trying to make the connection with slope. We're just going to say they went 30 mph and then came to a stop, then went 40 mph and came to a stop then went 20mph. Then they have to go, "OK which one was faster? 20 mph is the top of that line is the lower slope."

Monty: So they'll have to look at the graphs on the boards and say, "What is the slope of that?"

In addition to simplifying the lesson to meet the BMI by anticipating students' mathematical reasoning, they reformulated or removed some parts of their lesson so that only the essential elements were included that were necessary to meet the BMI. An example of this is how the group almost entirely removed the map from the lesson. Students would not need the map to find the distances and velocities in order to fill out the table. Additionally, the LSG decided to include a map with all five routes highlighted, but speeds would not be represented through color-coding and the routes would not be labeled. Though Amanda lobbied to keep the map with its information to allow students more ways of connecting the graphs with it, nevertheless, due to streamlining efforts the map became a source of context so that students could see hypothetical routes. Since, some parts of the tasks and materials were not critical to meet the BMI they were removed from the lesson.

Felicity: I think the map makes it easier and another ways makes it a waste. Because I think this, from the speeds to the graphs makes the connection that we want. If they have to go from the speeds to the map back and forth to the graph, what's the point? I don't see the point of having the maps.

Amanda: The point is that that is where we got the information, that's the connection we're trying to make that the graph comes from the speeds.

Felicity: I see that but I think they made that connection when they made their own graph. That's how they made that connection. I'm afraid if we use that it will handicap them from making the connection with the graph with the slope and the speed. I don't think it makes a huge difference but it just seems like more work we have to give them.

Later the LSG has an extensive discussion on the map and decides to use it as a source of context and not information.

Felicity: Are we giving them just the speeds or a map with the speeds? Because last time we gave the distances still. In that case they have the speed and the distance, basically everything. When we analyze do we give them a map with the routes marked or do we just give them the speeds?

Julie: I think all we need to give them is just the speed.

Amanda: I told Monty and Felicity, I think everyone will be able [to] make the connection with just the speed, well maybe that's all we want to do.

Julie: It would make it more real for them if they have a map.

Amanda: It would make it more real or maybe it would make…the connection with the graph.

Felicity: A happy medium perhaps for both of them, what if we give them the map? But instead of giving them a map that has all the speeds marked what if we gave them a map that was just like this that was just like, "Here is a route and here is a route." We don't give them the distance, maybe we still could give them the distances. Maybe that would give them the connection that they would be able to see that this is the route with 30 mph.

Monty: Here is one big positive I think. If it is just colors and has routes and then say here are the five routes, and they can say, "Oh route one is this graph. And I think it matches with this one. And the reason is that this one is steep and this one is a farther distance and this one goes the farthest." And so there are all these connections happening. This is a route kind of idea. But we won't mark each color. These are the five possible routes just so they know or have an idea.

Amanda: I remember in our class people were like, "We had no idea what routes that other people were doing. We thought everyone was doing the same."

Monty: What Julie was saying is maybe not have all the numbers on [the map]. But it would be beneficial to have an idea of what the other routes could be.

Felicity: So the routes, they're not going to have any speeds.

Julie: The thing I'm wondering is if our BMI if we only want them to focus on slope, they are going to focus on other things. Which is fine with me it is fine to make more connections. But we need to decide what it is we want them to do. If it is just slope, then we just give them the speeds. But there are other ways because they can easily…

Amanda: That's exactly the point I'm trying to make. If our main big mathematical idea that we want them to come up with is that slope is speed in this graph, then all we want to give them are the speeds.

Felicity: This is what I think it is. "You're life has mathematical data that can be graphed. These graphs shows that something is happening. We can use direction slope and graphs to interpret what is going on. We can also use these graphs to predict similar future events."

Julie: I think we should give them a map that just has the five routes in one color each.

Felicity: Like this but no different colors?

Julie: Just one color for each path, because that will solidify that…. I think that would be helpful for them to see it. But, we should tell them.

Monty: I agree 100 percent. I think each group needs to have a map with all five routes. That just gives them an idea of what the routes are. Not with any information besides that that is a route, you know they don't have speeds they don't have distances.

These examples exemplify the first three faces of ASR. The LSG refined their tasks by considering what mathematical reasonings or connections were possible given the materials, resources, and information that comprised the task, and then decided whether those possible mathematical reasonings were in harmony with the BMI. Sometimes they went back and forth between two elements on Figure 2, for example between the task and the reasoning it allowed in order to refine the lesson. If something was not right, whether the tasks and materials left open lines of reasoning that they wanted to shut off, or if it was not clear that students would be forced to examine the slope, then the LSG would work to remedy those issues.

Their dialogue portrays how they used their goals to shape the tasks, then anticipated the reasoning given the newly modified tasks, lastly they reflected on whether the tasks and associated anticipated mathematical reasoning would meet their goals. They went through this process with several other parts and ideas of their lesson such as, the graphs and the information about the routes which was cut down to sequential velocities. A major point of consideration in particular was the information about the routes to be given to all students during the matching phase of their lesson. Their efforts were for the purpose of directing students to their ultimate goal that they would recognize that slope represents speed in this context. In particular, at this point in their meetings they were working hard to streamline the lesson and direct students through their tasks to their goals. The pressures they faced probably bring the working operation to such a level of transparency that it can be represented with Figure 2.

The Fourth Face of ASR: Can They Do It? Does it Make Sense?

Refining the tasks and preparing the materials consumed almost all of the meetings the LSG had between teaching to their peers and teaching to the 9th grade class. Perhaps, because of this reason the fourth face of ASR is evident, anticipating students making sense and meeting the lesson goal. Since they were working so hard to refine the tasks and materials they often discussed whether students would make sense of it, sort of as a finishing stamp of approval. In one of their meetings two members showed up quite late, which allowed the other two, Felicity and Amanda, to draw graphs of the newly refined routes. When Monty arrived they had him work the task, deciphering which graph represented which route where the only information that he had was the sequence of velocities. This was on November 9, 2004, and was the fourth meeting since teaching to their peers; they had made most of their significant revisions by this time. Refining the routes had taken a great deal of work and had consumed much of the previous three meetings. Now they had the finished product to test. The following excerpt is from their discussion immediately following his completion of the task.

Amanda: What did you think about that?

Monty: Very doable.

Felicity: It's not too easy? You actually did the slope. Do you think anyone will actually do the slopes?

Monty: What's another way you can do it?

Felicity: What I'm wondering, I'm hoping students will do what you said students will start trying to look at the slopes. But, I also know from what our class did, they just kind of looked at it just as steepness, and they did this one didn't have a stop but they changed speeds.

Monty: I think that's ok. I think that is very applicable to our BMI.

Felicity: The question we had in general we didn't think they were different enough. They all seem so much alike, do you think that is good? Or bad?

Monty: I think and my reason is they'll have to understand what the difference is. So, there are differences and they'll have to figure out what those differences are.

Amanda: Is it easy enough to tell that those are two different slopes?

Monty: Easy enough?

Amanda: I mean, I would almost overlook that and think, I mean from this angle it looks, it looks the same, it just looks like there is a pencil mark there.

Felicity: And the thing is, that is the difference between 35mph and 55mph.

Amanda: So do you think the slopes are different enough?

Monty: Different enough by looking? But the point we are trying to get at, it will be realized what is happening here.

Amanda: Well what if the point wasn't there?

Monty: If you look hard enough you could tell.

Felicity: The slope changes there so you're going to have to notice.

Monty: If you paid enough attention you'd be able to tell.

Felicity: Because it's not a straight line.

Monty: You should be able to tell.

At this point the LSG moved on to discuss some other ideas not associated with this aspect of the lesson indicating that they were satisfied with their final project. The feeling among the group was that the task was finally designed so that students could only use certain lines of reasoning, and that it could successfully be completed. The theme that students would be able to successfully complete the task or the feeling that students would make sense of a task or question runs throughout their meetings and is highlighted in this excerpt by Monty. It seems to answer the question they might have of "Will students understand this?" and, "Will students be able to do this?" In this excerpt they talked about different aspects of the task such as the change in slope of the line being difficult to see, but they repeatedly stated that students would make sense of it and be able to do it. Remember their high level of confidence in their students reported earlier. Perhaps this high level of confidence in their students prevented them from seriously considering how students would come to understanding.

The Fifth Face of ASR: Anticipated Emotional Responses

The fifth face of ASR that was apparent in the data was anticipating emotional responses. This idea includes such elements as, motivation, realism, engagement, fun, students valuing their own work over that of a researcher, etc. Anticipating emotional responses was used to plan certain parts of the lesson. For example, the LSG anticipated that a particular group size would enable more students to engage in the task.

Monty: Are we going to give them the graphs on a worksheet?

Julie: I think if we did that, like that totally frustrated me as a student.

Amanda: The only thing I worry about, I think it is good to have their graphs on the board, but, I often have a really hard time seeing. I get discouraged if I can't see. Like if I have to put that much more effort into just being able to see the graph in the first place, then I'm not…

Monty: Do you get discouraged like that if you are working in a group? With peer pressure?

Amanda: What do you mean?

Monty: Because we are in your group are you going to be motivated enough to try. Because I feel the same way when I am working by myself and I can't see I'll just wait until I get an explanation kind of thing you know who cares. But, in a group when other people are depending on you do you think you have a little more motivation?

Amanda: Yes. I guess if we just stress that they should really get up and look at them.

Monty: I think we have an exceptional class too, with the students like Felicity said.

Felicity: Are we having them analyze in partners or groups? Because to me that makes a huge difference. With what you're saying with motivation if I'm in a group setting with four of us I'm not going to be doing any work. If there is two then I'll be OK I can just sit here and let my partner do everything. Are we doing it in partners.

Julie: I prefer partners.

Amanda: So we're doing the graphing in a group but the analyzing in partners. (Pause) We just need to make sure it is big enough.

Julie: That's what I was thinking. Some of the girls in there are so shy would they really get up and go look?

Felicity: Let's ask Mrs. Peabody that when we go see her.

Julie: Because I wouldn't when I was in Junior High. I was super super shy.

Notice how they considered their own emotional responses and projected these feelings to how the students might respond. In the excerpt above, much of the discussion centered around Amanda's concern that if students do not have easy access to examine the graphs they could become frustrated. Her concern spurred further discussion about group size and logistics of the lesson that would maintain student motivation.

The LSG also anticipated emotional responses when they talked about how students would value and understand their own graphs over the same graphs provided by the researchers.

> **Monty:** About using the students [graphs] to do the analyzation. [We should] try to stay away from giving them graphs that we made. Just because I think it's so much more impactful, that a student, "My peer made this graph."
>
> **Felicity:** I was also afraid that when we ask them to make this graph, this happened to me a couple of times in high school, we go to the work of making this graph and then the teacher busts out there own that they've made. And it's like, "Thank you for doing all this work but I did it right."
>
> **Julie:** That's true.
>
> **Felicity:** So let's just stick with having them graph.
>
> ….
>
> **Felicity:** The negative aspect is that it is saying to the student that their work doesn't really matter because we already did it for them.
>
> **Julie:** It is just busy work.

These two previous examples illustrate how the LSG considered feelings or emotions to shape the task to better reach their goal. In the excerpt above some of the underlying dilemmas that the participants were dealing with was how students would get access to the graphs, and the possibility that if students graphed incorrectly they would not be able to match the graph with its corresponding route. Both of these issues were resolved and in this excerpt were minor to the emotional impact, the feeling of ownership of the task, that was anticipated if the students created the graphs themselves.

Summary of the Five Faces of ASR

The nature of the anticipated student responses by this preservice mathematics LSG was centered on defining and refining the task. Although the five faces described here have been exemplified by dialogue from the LSG's meetings between teaching to their peers and teaching the public class, they were also present in their discussions prior to teaching to their peers. The anticipated student responses that were brought up were often spoken through a student perspective, and were in the context of thinking through the task design for the purpose of defining and refining it. While there were many ideas that their anticipated student responses focused on, there were at least five different frames of anticipating student responses that were specifically used to craft the lesson. These five frames or faces seem to answer five different but related questions in the preparation of a lesson:

- Will the task or overall lesson design guide students toward meeting the lesson goal? This correlates with anticipating toward the lesson goal or BMI.
- What mathematical reasoning might students use to perform the task and is it acceptable? This correlates with anticipating the mathematical reasoning that students could use.
- Given the first two questions, how can the tasks be designed with materials or information or resources to better meet the goal and provide a more ideal pathway or pathways of reasoning to meet it? This correlates

with anticipating whether materials, resources, information or lesson parts are consistent with the goals and mathematical reasoning that is desired.

- Will students make sense of the lesson as it is presently designed? This correlates with anticipating whether students will be able to work successfully toward the BMI.
- What emotional responses might students have to the lesson and how can the lesson be more effectively designed to engage students? This correlates with anticipating emotional responses.

These questions are central to designing tasks. Having tasks fairly well designed and ready to be implemented would enable teachers to then consider two more global questions about lesson design proposed by Wiggins and McTighe (1998). Their questions were:

- What enabling knowledge (facts, concepts, and principles) and skills (procedures) will students need to perform effectively and achieve desired results?
- What will need to be taught and coached, and how should it best be taught, in light of performance goals?

Whereas the questions that participants in this study appeared to be trying to answer are more focused on designing the individual pieces of a lesson, Wiggins and McTighe's questions are more general. Their questions focus on prior knowledge, and given the tasks deciding what parts of the tasks will need to be more strongly supported by the instructor. However, Wiggins and Mctighe still do not ask the question, "How are students going to come to an understanding of the content?"

Corroborating Data

The video data provided a wealth of information regarding anticipated student responses because anticipated student responses that were used to design the lesson were not reported in the final lesson plan. Additional data, including pre and post interviews, individual lesson plans, journal prompts, and the final lesson plan of the LSG provide little additional information about the nature of ASR from the participants' perspective because the out loud thinking that made up their anticipated student responses was not captured. An observation worth reporting is that the rich discussion that occurred as the group met expanded greatly on the skeletal information provided in other data sources. For example, the individual lesson plans prepared by the students tended to have right and wrong possible answers with some mathematical reasoning in the ASR column. In LSG meetings the participants expounded on right and wrong answers by discussing both rationale and potential consequences.

What was found in other data sources serves to support the findings in the video data of group meetings, namely that their anticipations were in relation to task design. Amanda responds on November 14, 2004 to the journal prompt: "How has the teaching of your research lesson to your peers impacted your research lesson?"

> I felt like just being able to teach the lesson gave us a better idea of how a student might interpret our instructions. Our group changed a few things that we felt didn't go over very well, or if we felt that it didn't really address our Big Mathematical Idea.
>
> We decided to just give a list of the speeds when they were trying to match the routes to the graph because we wanted to focus more on the

connection between slope and speed. We also felt that it was important that we create the scale for the graphs, because even our class would have a hard time analyzing the graphs if they aren't in the same scale. I feel like our experiment teaching was a good experience. It helped me feel more confident that our lesson has potential to reach students of all different mathematical levels.

In the initial and final interviews, several of the members of the LSG indicated that they initially thought of ASR in terms of right and wrong answers and misconceptions. This superficial understanding of ASR could explain why the LSG generally anticipated student responses relative to task design. However, in post interviews when participants were asked what ASR meant to them, what things they anticipated, and how their LSG anticipated student responses, three of the four participants responses indicated that ASR was more than task design. Several went on to say that they spent a great deal of time anticipating toward task design, but that there was another type of ASR that they had not done.

Julie: We didn't like analyze their methods so much of how they would figure things out, but more how they would interpret things. Like Monty asked a question one time that I was confused on and I was like, "They might think of it this way." And he was like, "Oh that's true. I didn't think of that." Or, like when we were thinking about how they would analyze we were like, "Well they might just ask their friends what graph they did and be like, 'Oh you did it that way.'" And that's why we wanted them to tell us why they thought each graph was whatever which one. So we like anticipated that. We kind of anticipated that in the filling out the tables they would either use the formula or proportions, but we didn't go much deeper than that, which we probably could have. But, yeah, we didn't do a deep thought process about that. Just kind of surface level.

Both Julie in the quote above, and Amanda in the quote below speak quite clearly that they were using student responses to guide the flow of the lesson which meant that they worked to refine the tasks.

Amanda: I think we were just trying to get our point across so strongly that we…didn't want them to…mess up or something. I don't know. You know we cared so much about making our point and we were trying to be so careful about wording everything correctly and making sure that what we were saying was clear so that their answers could be clear and you know the thinking would be natural and easy for them to think through. You know and I think that's good to do, I think that's good to do. You know I think where we really failed to anticipate their responses was on the table. We should have spent a lot more time thinking about that and anticipating responses there. You know going through different ways to solve it. Going through different way to think about it and working those together and thinking about those.

Monty describes the difference between the two types of ASR in terms of the motivation for each.

Monty: Well, I've noticed that I have problems with all my anticipated responses or ways that I thought. So I need to try to really understand the

> students, you know…[having] knowledge of your student and anticipate what they think. So that's been a big step for me to try and think what would they think rather than what are different ways that I can do the problem. You know what I mean? I think there's a difference between that. Anticipating student responses is different than all the ways you as a teacher can think of the ways that you would do it that the students might do. But that's not the same thing….You might get the same anticipation, but yet it's different because you're, you're motivation is different. Your motivation is so you can be a good teacher on that first, on that one. The other one is concern for the student.

Summary of the Data and Analysis

The BMI played a major role in the development of the lesson prepared by the LSG. It served as a focusing and filtering mechanism in the group's discussions to design the lesson. The LSG anticipated student responses to determine if a suggested idea or refinement would enhance, simplify, or focus the lesson so that students would arrive at the lesson goal. The object of their anticipations was the details of the tasks and the motivation of their ASR was to fine-tune the tasks and lesson plan so that they would run smoothly toward meeting the BMI. There are at least five different faces of ASR that were used to design the tasks. It is not known if the participants could have anticipated students' conceptual development, but evidence suggests that they understood the difference.

One of the possible reasons that the LSG did not anticipate at a deeper level is because of the great detail that had to be attended to in designing and refining the tasks. A question that cannot be answered by this research is whether the participants had the ability to anticipate the mathematical conceptual development. Excerpts above suggest that the participants were aware of this type of ASR, even though they did not anticipate student responses in terms of their students' conceptual development. Questions they might have asked themselves that would have helped them anticipate students' conceptual development is, "How will students come to understand this idea? What will indicate that they do understand? How will we know what they are understanding and what they are not understanding? How will the concepts develop for the students given the lesson plan?"

Not all moments in the LSG meetings noted as ASR fit into task design. Moments in the data that fell outside of task design were few and brief, making it difficult to clearly identify their nature and purpose. One item discussed in the first half of the analysis that appeared more clearly than others was the LSG anticipated student responses to inform themselves of general lines of reasoning that students could use. While there were several instances where they voiced ideas that were part of students' conceptual development, they did not take the opportunity to collect, examine, and discuss those ideas in ways that would enhance student understanding. For example, Monty described how students would figure slope on a segment of a graph to find speed, but they did not discuss how students would think to do that, and how the teacher might support them doing that.

Conclusion

Overview

This research was conducted to better understand ASR as it is characterized by preservice mathematics teachers in a lesson study group setting. It is hoped that the findings presented here will enable teachers to better understand ASR and its value in improving lessons. The concept of anticipating student responses or thinking is alluded to in a great deal of the literature on teaching and ASR is briefly discussed in much of the lesson study literature. This research has been conducted to increase the understanding of ASR, particularly in the lesson study environment.

ASR fits well in the quasi-scientific method approach to lesson planning and design. ASR serves to generate the hypotheses that are tested in actual classrooms. There appear to be at least two fundamentally different but important types of ASR. First is the conceptual development type of ASR inferred in the literature. This type of ASR is a mechanism by which teachers begin to carefully examine student thinking and prepare to orchestrate discussions of the concepts. The second type of ASR as described in this research is about task design for the purpose of creating tasks and lessons that are engaging, that students can successfully navigate, and that meet lesson goals. It is hypothesized that this second type of ASR is a natural and necessary precursor to anticipating students' conceptual development.

Five different faces of ASR were evident in the data. They served to help the participants develop the lesson and tasks by addressing major issues of lesson design. These questions again are:

- Will the task or overall lesson design guide students toward meeting the lesson goal?
- What mathematical reasoning might students use to perform the task and is it acceptable?
- Given the first two questions, how can the tasks be designed with materials or information or resources to better meet the goal and provide a more ideal pathway or pathways of reasoning to meet it?
- Will students make sense of the lesson as it is presently designed?
- What emotional responses might students have to the lesson and how can the lesson be more effectively designed to engage students?

After lesson researchers have created a task and have used ASR to shape it as they address these questions, then they can begin to hypothetically experience the lesson from the student's perspective. In other words, this is when the researcher anticipates the understanding that is necessary for a student to have, how that understanding will develop, and how it can be enhanced. This is where the ordinary teacher comes to know what really makes for the learning of a concept for a class of students. However, one cannot perform this activity without having a coherent lesson designed to meet the lesson goals. So in the final stages of planning a lesson, after the lesson design is more or less in place, both anticipating student responses for fine tuning of the lesson and anticipating students' conceptual development occur hand in hand.

Perhaps it is natural to anticipate task responses not only because it is antecedent to anticipating conceptual development, but also because it is easier. We have a better ability to use language for superficial issues. It is easy for us to talk about how long something took, whether something was misunderstood or misinterpreted. It is almost effortless to say something is easy or hard, or fun or boring or interesting, or that somebody achieved the same answer with a different procedure. Perhaps it is easy to describe these issues, because they have nice key phrases that capture them that are used in our everyday language. If we want to look deeper into conceptual development, then we have to be intelligent, knowledgeable about the concepts, and sensitive to learning as it occurs. We have to struggle with the everyday language to describe things we don't talk about everyday. Empson and Junk (2004) wrote, "There is a great deal of intellectual work involved in learning to hear and to understand the mathematical significance of children's thinking that goes beyond acquiring the specifics of knowledge of mathematics" (p. 140). It is simply easier to describe symptoms, than to take the time to explain precisely what the matter is.

Creating the lesson posed significant difficulty to the participants such that few opportunities to think critically about students' conceptual development arose or were noticed by the participants. When someone felt inclined to reflect more deeply on the mathematics, others were worried about task details. The participants valued students' responses and thinking, but had difficulty creating a lesson that in design was sufficiently restrictive to draw students toward the lesson goal and at the same time was sufficiently open-ended to allow students to travel their own mathematical pathways. Simon (1995) describes this as the "creative tension that shapes mathematics teaching" (p. 141). Participants fit in the level 2 or level 3 engagement with children's mathematical thinking as described by Franke et al. (2001) (see Table 1). They believed that students could solve problems without explicitly being taught and provided opportunities for students to discuss their thinking, but they did not discuss how students would come to understand the concepts involved in solving the problems they posed. They were aware of the general strategies students could use, but did not use their knowledge of students thinking to drive instruction.

Limitations

In designing the different tasks and the overall lesson, the participants spent a great deal of time considering the different mathematical reasoning that students could employ in order to streamline the lesson toward the BMI. It is likely that the enormity of this labor inhibited the participants' engagement with student thinking at a conceptual level. Furthermore, the participants in this study used the phrase anticipating student responses, and while their appears to be evidence that they understood ASR to include anticipated student thinking, the phrase ASR may very well have restricted them from thinking clearly in terms of or focusing on student conceptual development or thinking.

It must be remembered, too, that the participants' were preservice mathematics teachers from the U. S. where procedurally oriented math classes are common (Stigler & Hiebert, 1999, p. 10), and their participation in classes like this may have made it difficult for them to think of the students' conceptual development. In lesson study in Japan, the research lesson is but a subset of a unit, allowing flexibility on both ends. Ideas or activities can occur in prior lessons that are used in the lesson, and the lesson does not have to have an end all discussion on the topic. Follow up discussions, concluding discussions, or building on discussions can occur afterward. Since the participants in this study did not have these options they felt the need to have the whole lesson in one class period. Therefore, it is possible that the findings of this research are disproportionately skewed toward task creation and modification. The benefit of this, however, is that the data provide detailed information of what experiences may be had when creating a lesson from scratch. Therefore, the data allow careful examination of anticipating student responses toward task design than would otherwise be possible to report.

The literature indicates that teachers in Japan who engage in lesson study rarely design a lesson from scratch, but that they often use many experiences, examples, resources, and lesson study reports to create initial lesson plans (Chokshi & Fernandez, 2004; Lewis, 2002b). What has been uncovered in this study may explain why Japanese teachers do not design a lesson from scratch. This enables them to more quickly do the rough framing of a lesson and consider student responses and thinking at an in-depth conceptual level. Nakano (2002) relates that:

> What we learn from lesson study changes as we accumulate experiences. When you have little experience, you learn methodology and how to run the class…As you gain experience, rather than learning how to conduct the class, in prelesson study you can get to know the value of mathematics and the value of the materials. (p. 66)

In researching ASR through the experience of preservice teachers who are at first unfamiliar with lesson study, it is acknowledged that how they understand and use and generate anticipated student responses will not necessarily be similar to that of experienced teachers who are accustomed to lesson research. Nevertheless, it is believed that ASR can be a powerful source for teacher development because ASR is a window through which all teachers can begin to engage students' conceptual development.

Two additional factors that may have limited the experiences of the participants, which in turn may have affected the findings of this research, are the BMI and the observational component. As described, the group initially had quite a broad goal but few specifics. This began to concern them because they felt a need for more specific direction. Then the goal became quite narrow, and great effort went toward creating a lesson where students would learn that slope represents speed. Those familiar with lesson study goals would likely suggest, and researcher concurs, that the LSG describe in detail the mathematics that students might use to make the connection they sought.

The observational component was not very well developed among the participants. This may be due to a weakness in the methods course. Students were taught to find as many different ways of thinking as they could when they observed lessons and were taught that listening to students' reasoning was essential to teaching open format lessons and orchestrating accompanying discussions. However, there are many things that good teachers need to be able to observe in evaluating a lesson that the participants were exposed to due to the attached observations course. And little time was available in an already full curriculum to thoroughly discuss or elaborate on the purpose of the *evaluation/misc.* column that the participants used in the lesson plan format provided for them (See Appendix B). Had the observers been required to routinely describe what they had observed in the methods class, the instructor may have been able to better direct their observations toward students' conceptual development.

In examination of the final research lesson plan created by the LSG, the evaluation/miscellaneous column was entirely devoid of evaluation type notes. Remember that, "Japanese teachers stressed [the need] to think carefully about what evidence was required to claim that a lesson had achieved its objectives" (Fernandez, et al. 2003, p. 175). It was observed that participants had no such preconceived evaluations and while they were observing their lesson to the 9th grade class they seemed satisfied to note that students were successfully working. A final note here is that lesson researchers speak of lesson study as a means to develop the eyes to see their students. In light of the feeling that lesson study is a slow process of gradual improvement it can be construed that learning to see with such intelligent eyes is not easy. This brings to mind the concept of the expert blind spot discussed by Nathan and Petrosino (2003). In their research it was found that, "participants relied largely on impressions in making their difficulty ratings, rather than on [a] more deliberative, analytical method" (p. 919).

Beliefs play an important role in what one does and what one looks for, and what one hopes to do. This research was not designed to examine or discuss in depth the beliefs held by the participants. However, some strong indications as to participants' beliefs appeared through the data collection and had this been an integral point of interest in the methodology it may have shed more light on their conception and use of ASR. In particular, all participants expressed the desire to teach after the reform manner, and several of them further expressed the concern that there may be little or no support in their work communities to do so. Yet having expressed these beliefs does not provide a clear picture of their belief system, making it possible only to conjecture how their beliefs affected their thoughts and actions. It should be noted that occasionally participants made statements about teaching beliefs that later appeared to be contradicted by their actions. Therefore, it is likely that while they had come to believe certain things about teaching and learning, these beliefs had not been fully synthesized.

Implications and Future Research

ASR is an impressive tool for the shaping and refining of tasks, questions, materials, and overall lesson design. Where the literature reports the aggressive use of anticipation to come to know how students' ideas develop, ASR can be used just as aggressively to create a lesson from scratch and refine it.

Teacher educators, wishing to initiate teachers to lesson research and familiarize them with anticipating students' conceptual development might consider the following suggestions:

1. Encouraging very detailed lesson goal(s). Having a clear description of the mathematics to be learned enables researchers to begin talking about how it will be learned. If the lesson goal speaks only of generalities of the ideal student, the researchers may have difficulties in identifying essential details of students' cognitive activities. A descriptive lesson goal that includes thorough explanation of the content to be taught is likely to lead researchers more naturally and quickly to anticipating students' conceptual development. Likewise, anticipating student responses in depth is likely to lead to researchers having better eyes to see learning. A strong observational component may very well be the end of the cycle of anticipating student responses that loops directly back to ASR. Therefore, a well developed sense of what observation is about, may be one of the most powerful access points into student thinking and anticipating student responses.
2. Teaching the importance of observing very carefully the development of understanding in the classroom. Participants need to know that when observing or

evaluating a lesson, the purpose is to examine how students learn and to gather evidence of their learning and how it came about.

3. Encouraging multiple (greater than two) iterations of the lesson study process. This way, if the lesson has undergone major revision, by the third and fourth time through the participants will be able to carefully examine student responses and how their understanding develops as a result of their lesson. It is hypothesized that if the participants of this research were to teach the lesson one more time, they would not have had to refine the details of the task but could have instead focused on student responses and learning. If they had taught the lesson one more time they would have been exposed to a diversity of responses which may have promoted discussion about students' conceptual development. Christensen and Hooker (2000) discussed how anticipation gets better with experience. They wrote:

   > Anticipation itself can form a very important aspect of evaluation because a system can evaluate whether its anticipations are correct, as measured by whether they yield successful action completion. This provides a means for the system to generate new information both predictive and normative information, about the conditions of successful interaction. (p. 16)

   Therefore, multiple cycles through the lesson study process, for those new to lesson study, are likely to enhance participants' awareness and understanding of student responses.

4. Encouraging the use of or providing experienced others or multiple resources. Having multiple resources is likely to aid those new to lesson study as well as aid those familiar with lesson study. Though those familiar with lesson study do not always examine previous research lesson reports, it would be helpful to those unfamiliar with it. In particular, had the researchers been given a task to design a lesson around they would have been that much further along in their planning which would have given them time to consider student conceptual development. Shulman's (1986) call for case studies that combine both content and pedagogy has long been answered in Japan by their research lesson reports. A similar collection could be extremely valuable to U. S. teachers and could easily be collected in an on-line database.

While ASR was used well to streamline and refine the lesson reported in this research, the participants seemed to tire of continually modifying the lesson to more ably meet their lesson goals. In their reflection meeting, several expressed a desire to examine more carefully the mathematical reasoning that students used. Perhaps this is because conceptual development is more fascinating than task design, just as an experiment is more engaging than the proverbial drawing board.

Future research into the concept of ASR could be enhanced by including a thorough examination of the participants' lesson observations, what they observed and why, and similarly an examination of their beliefs and the questions that generated productive discourse. All may shed further light on ASR itself, and furthermore situate particular anticipations in the context of particular participants. It is hypothesized that

ASR coupled with observation of students' conceptual development is likely to alter participants' belief systems in ways harmonious to reform efforts.

In summary, anticipating student responses is a remarkable tool that is accessible to all teachers that can be used to refine lessons and understand students' conceptual development in the classroom. By anticipating student responses teachers can continually refine their lessons and their practice of teaching to be more sensitive to the intellectual needs of their students and more capable of enabling students to learn. Krainer (2004) wrote recently in the editorial of the Journal of Mathematics Teacher Education:

> Teachers have to balance mathematics teaching between two extremes, loading the students on the back of a truck and driving them at high speed along mathematical highways…or leaving them with a pick and a shovel and the task of building their mathematical paths themselves…Consequently teachers are mediators between …envisioned target knowledge…and students' existing prior knowledge of mathematics. (p. 88)

ASR is useful to scaffold, enable, and streamline lessons in partially uncovering the mathematical highways, and also, to understand the journey that students are likely to experience along the way. In this way, ASR allows teachers to act as architects, builders, and travelers of mathematical highways.

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Appendices

Appendix A

Lesson Plan Template

Teacher Activity	Anticipated Student Thinking and Activities	Points to Notice and Evaluate	Materials, Strategies

Note. From "Lesson Study: A Handbook of Teacher-Led Instructional Change," by C. C. Lewis, 2002, p. 129.

Appendix B

Lesson Plan Template for Methods Class

Math Ed 377 Lesson Plan

Lesson Body

Lesson Sequence: Learning Activities, tasks and key questions (What you will say and what you will ask the students to do.)	Time	Anticipated Student Responses	Teacher's response to student thinking/reactions. Things to Remember	Evaluation/ Misc.

This extends for several pages.
At the end is this.

Whiteboard Layout

Appendix C

Chronology of LSG Meetings

Date	**Duration (min)**	**Brief Description of Meeting**
September 23, 2004	20	Group meeting with public school teacher.
October 1, 2004	11	Brainstorming for BMI.
October 4, 2004	65	Brainstorming for BMI.
October 7, 2004	110	Creating the lesson tasks.
October 11, 2004	44	Working out details of the lesson tasks.
October 12, 2004	105	In-depth work on lesson tasks and creating the materials.
October 13, 2004	52	Walk through and discussion of the lesson.
October 13, 2004	45	Teaching the lesson to their peers.
October 22, 2004	32	Reflecting on the lesson to their peers without Monty.
October 26, 2004	34	Reflecting on the lesson to their peers without Amanda.
November 3, 2004	56	Creating a fifth route and major discussion about how students will analyze.
November 9, 2004	93	Trying out the tasks and fine tuning lots of details.
November 11, 2004	10	Meeting with public school teacher, getting some questions answered.
November 29, 2004	60	Talking out the lesson plan.
November 29, 2004	25	Final preparation of materials.
December 2, 2004	60	Lesson to public students.
December 6, 2004	40	Discussing final write-up prompts.
December 6, 2004	75	Class reflection meeting.
December 13, 2004	20	Meeting to edit the final write-up paper.

Appendix D

Initial Interview Questions

Journal prompt questions

1) In the first journal prompt you said "________" about what you would observe, can you tell me more about that?
2) How does the way that you learn math best affect how you teach it?
3) What general types of feedback do you tend to give a teacher after their lesson?

General questions

1) What kind of things do you consider when you prepare a lesson? Why are these things important?
2) When you prepare a question or a task what kinds of things do you think about?
3) How do you create a task or question that will help students learn?
4) What kinds of comments do you tend to make after a lesson and why those comments?
5) When you observe a lesson what is important to observe? Why?
6) How does your knowledge of your students affect your lesson plans?
7) What makes a lesson successful?
8) Tell me about the lesson you taught and why you made it the way that you did? How would you do it differently having taught it already?
9) In your own words describe what a group does in lesson study?
10) What do you think it means to anticipate student responses?
11) What do you get out of anticipating student responses?

Treasure task

1) If you were to give this task to your students what thoughts, ideas, or connections might they make?

2) How might a ninth grader represent the solution to this task?

3) Can you think of any other representations they might make?

4) Why do you think these solutions are likely?

5) How do you imagine a ninth grader will respond to this?

6) Tell me everything you imagine will go through a ninth graders head when they get this problem.

Appendix E

Final Interview Questions

General questions

1. Has your conceptions of how to teach changed because of this course? How? Why?
2. How does your knowledge of your students affect your lesson plans?
3. What do you think it means to anticipate student responses? And, How has this concept of ASR evolved over the course of the semester?
4. When you prepare a question or a task and you ASR for it, what kinds of things do you think about?
5. Is there something that you felt should have been done better or differently or simply been done that was not as you met in your group?
6. Why did you choose to write up your lesson plans for the group lesson separately? How did this affect your lesson? How would it be different if you wrote it as a group?
7. If the way someone filled out the table was not significant (it wasn't part of your BMI), then why have groups fill them out? What is the rationale for having groups fill out the table? If it was to help students connect real-life with math then how do you know if it did that?
8. Tell me how your group anticipated student responses?
   - Would you have preferred to do anything differently?

Task Questions

1. Solve the following problem.
2. Anticipate student responses for this task.
3. Compare and contrast these student solutions.

For Amanda:

1. You said that the graphs looked too similar during one of the sessions. What did you mean and why was it significant?
2. You wanted the first rate to be 20mph or 30 mph, why and why did you want the second rate to be 25 or 35 or 45?

For Monty:

1. Do you agree or disagree with Dr. Peterson's philosophy of teaching? How is your philosophy different than his?
2. You mentioned in one of the last group meetings before you taught Sophia's class that when the students were in the analysis part of the lesson at the end that you hoped that there would be discrepancies between what students thought and that they would argue over it. Why did you want that to happen? Of what value would it add to your lesson?
3. You were overheard to say during the lesson in Sophia's class that filling out the table took longer than you anticipated. Why do you think that was so?
4. Use a proportion to solve 35 mph for 2 miles. How much time does it take?
    a. How does 2/35 = x/60 make sense?

For Felicity:

1. In your journal prompt you said that you had actually ASR a lot, but not in a formulaic way, What kinds of student responses did your group mostly come up with? What other categories of student responses that may have been useful?

For Julie:

1. It seems as though it was very important for you to have all of the materials prepared before you thought about the lesson? Is that true? Why?

Appendix F

Journal Prompts

First Day of Class

- How is math learned best?
- What is mathematics?
- What do you look for when you observe a mathematics lesson?

Prompt Given Second Week of Semester

- When you are observing a student working on a math problem to find out what they are doing with it and what they are thinking, what do you look for or ask them?

Prompt Following Research Lessons Presented to Peers

- How has the teaching of your research lesson to your peers impacted your research lesson? Explain.

Prompt Given Just Before Lesson Study Groups Taught Research Lesson to Public

Journal Prompt: Due next Tuesday November 23.

Write one to two pages about anticipating student responses.

Following are a list of ideas you may want to consider:

- What does it mean?
- In what ways have you or your group anticipated student responses?
- How has your concept of anticipating student responses evolved?
- How is it useful to anticipate student responses?

Appendix G

LSG Final Lesson Plan

Lesson Plan

Big Mathematical Idea	Graphs are a visual way to represent data that we collect. Graphs show us that something is happening. They display a relationship between two variables, and we can use the slope of a graph to help interpret that relationship. Specifically, in a graph of time and distance traveled, the slope will represent the speed.	
Why is this topic Important	It's important to make connections between graphs and real life. Examples include sports statistics, or graphs that represent earthquakes or natural disasters.	
Unit Overview	**Prerequisite Concepts**	Formula for slope (y1-y2)/(x1-x2). Basic algebra. General understanding of mph, distance, and time. Proportions or formula distance = speed x time.
	Subsequent Concepts	More graphical connections. Deeper analysis.

Lesson Body

Lesson Sequence: Learning Activities, tasks and key questions (What you will say and what you will ask the students to do.)	Time	Anticipated Student Responses	Teacher's response to student thinking/reactions. Things to Remember	Evaluation/ Misc.
Split the class into 5 groups and hand out packets. Read the blurb about going to BYU.				Materials: Packets: (5) rulers map explanation/table directions.

Have the groups fill out the provided tables. Tell them to put time in minutes. Walk around while groups fill out tables. Call class back together when it looks like everyone is wrapping up.	10		Time in minutes.	
What were some struggles you ran into while filling out these tables?	5	Time in Minutes. Problem getting time from distance and speed. Figuring in the stop-lights. Some probably use formula D = S*T. Some use proportions.	How did most of you figure out the time? (Have students share their different ways) Give a few minutes to fix mistakes if needed.	
Now that we have all this data, how can we visually represent the info in our tables? What would make sense to put on each axis?	5	Prepare for blank stares. Come up with time and distance together.	Suggest a graph. Hand out pre-labeled graph paper.	Materials: pre-labeled graph paper.
Give groups time to graph their data. Put graphs on the board. Pair students off in partners. Hand out speeds and maps.	10			

Explain: Now I'm going to give you this map, which shows all five paths, but only the routes and not not distances or speeds. I'm also giving each pair a list of all the speeds each route traveled. Your job in partners, is to decide which graph (labeled A-E) goes with which route (labeled 1-5). You also need to explain why your answers make sense.	10	Asking other groups which graph is theirs, or using colors of markers they saw.	This is why we want them to justify why. Tell them they are free to get up and go to the board to see.	
Call class back together and start listing each possible answer people came up with for each graph.			Answers: A--1, B--4, C--5, D--3, E--2	
To determine the correct answers together, have students justify their reasoning to the class, at the board.	10			
Questions for further analyzing:				
What is happening at these flat lines?	10	They're stop lights, one minute but no distance.		
What is happening between the flat lines?		Traveling.		
Which route would you take to get to [the destination] in the least amount of time?		Use minutes axis and the end points of the graphs.		
Which route would you take if you wanted to spend the longest amount of time?				

Which path travels the shortest distance? The longest distance?		Use miles axis and the end points of each graph.		
Which path travels at the highest speed on any interval? The slowest speed?		Use steepness of slope--steeper means faster.		
What is the connection between distance and time?		speed	Use the graph to point this out. (Maybe specific example: use two points on graph A to calculate slope and show that it is the speed)	
What is the connection between slope and speed?		They're the same.		